# Loss aversion or preference imprecision? On WTA-WTP disparity without endowment effect*

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**Abstract**

We propose a setup to account for two leading explanations of the WTA-WTP disparity: one based on the loss aversion and the other based on preference imprecision. We propose two axioms that allows us to distinguish the part of WTA-WTP disparity atributed to each of these two explanations. Our approach is general and incorporates some of the leading models as special cases. To illustrate our approach we propose a simple experiment that allows to quantitatively decompose the WTA-WTP gap in the two analyzed channels.
**Keywords:** willingness to accept, willingness to pay, uncertainty aversion, loss aversion, incomplete preferences, short selling, reference dependence
**JEL classification:** D81, D91, C91

## 1 Introduction

The existence of a large difference between WTP and WTA known as the WTA-WTP gap), is one of the most widely discussed effects in behavioral economics. In this paper we are interested in the disparity for uncertain prospects, abound in finance, insurance, sports betting and gambling, for which the median WTA/WTA ratio is approximately 2 (Horowitz, 2006; Eisenberger and Weber, 1995).

The size of the difference varies depending on the design and elicitation techniques. However, there is a consensus that the gap is too big to be explained by the standard utility theory, which ascribes it only to the wealth effects arising due to the differences in initial positions in the WTA and WTP elicitation tasks. The prevalent behavioral explanation is based on the asymmetric treatment

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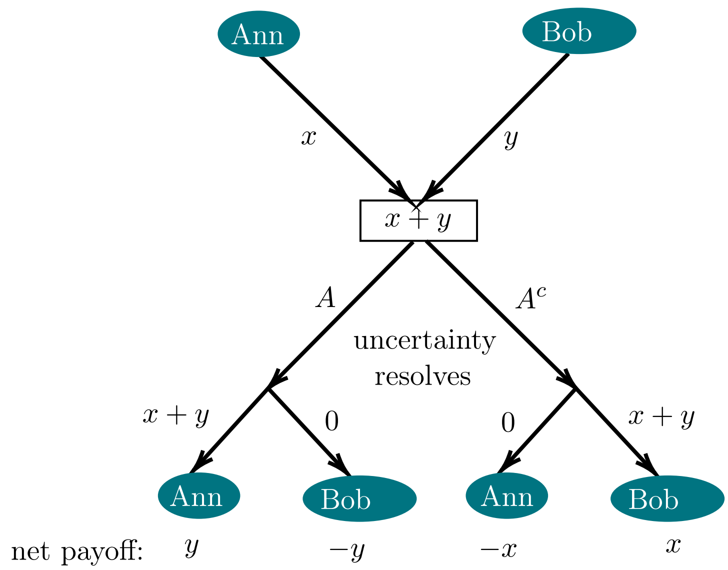
of gains and losses (Kahneman et al., 1991; Marzilli Ericson and Fuster, 2014). Losing the prospect is more painful than the joy of gaining it. It is challenged by the recent results of Chapman et al. (2023) who found that the disparity between WTA and WTA is only weakly correlated or not correlated with loss aversion. This has sparked a new interest in explanations based on preference imprecision or caution (Dubourg et al., 1994; Cubitt et al., 2015; Cerreia-Vioglio et al., 2015, 2024). Even though these explanations differ in the modeling assumptions and setups, they share the key idea, which can be summarized as follows: If the decision maker is unsure of the tradeoff he should apply (and thus unsure of the attractiveness of the prospect), he may want to buy cheap and sell dear just to be on the safe side. This formally translates into having a set rather than a single utility function to evaluate the objects. This creates discrepancy between WTA and WTP due to the opposing positions held by the buyer and the seller (the buyer wants to pay as little and the seller – as much as possible).

Even though Cerreia-Vioglio et al. (2015, 2024) are complete preference models, the same idea is present in their incomplete preference counterparts (Dubra et al., 2004; Ok et al., 2012 for risk and Galaabaatar and Karni, 2013; Hara and Riella, 2023; Borie, 2023 for uncertainty). The difference between the former and latter models lies in the definition of caution. In Cerreia-Vioglio et al. (2015) the DM is cautious in evaluating a certainty equivalent of a trading position. In models of incomplete preferences caution is a form of inertia when moving from the status quo:[1] the decision maker has to be sure that he likes the new option at least as the status quo in order to accept it. (Comment on cautious completions Gilboa et al., 2010; Hara and Riella, 2023) The advantage of incomplete preference models is that they deliver more information on preferences. Instead of forcing choice from the decision maker, we allow preferences to distinguish between indecision and indifference as in Cubitt et al. (2015); Agranov and Ortoleva (2023).

WTP and WTA are usually associated with the task of buying and selling. These tasks differ in the DM's initial endowment, with the seller initially owning a good and the buyer not. In the classic utility theory, this creates income effect which is the only source of the WTA-WTP disparity. The extension of this idea in behavioral economics is the endowment effect, in which owning a good changes the way you value it. This has led many to view the WTA-WTP gap as equivalent to the endowment effect. However, the evidence for this effect is weaker than that of loss aversion or WTA-WTP disparity (see for example Plott and Zeiler, 2005 or Marzilli Ericson and Fuster, 2014

[1] Bewley (2002) advocates the inertia assumption: "if a new alternative arises, an individual makes use of it only if doing so would put him in a preferred position."

Figure 1: Playing a bet ($x, y$ denote some postive money amounts)




for the survey treatment). In his experiment, Brown (2005) found evidence of loss aversion, but resulting not from the loss of a good, but rather from the negative net result of the transaction of buying or selling. We thus define WTA-WTP gap without the endowment effect. Instead of the seller of the prospect,[2] we can think of two opposing parties exchanging uncertainty:

- a lottery authority issuing the lottery ticket to the buyer
- a casino betting with a gambler
- a financial institution that hedges the risk of a person or enterprise

This leads to WTA defined as a **short-selling** – rather than selling – **price** of a prospect (Eisenberger and Weber, 1995).

As the payoffs of the buyer and short-seller are exactly the opposite and the status quo is the same in both cases, the WTA and WTP elicitation tasks force the agent to reveal his attitude towards gains and losses. We can thus ask the question – which of the two explanations, based on loss aversion, or based on preference imprecision/caution, drives the WTA-WTP disparity.

**Motivating example** To illustrate our approach, we start with an example of Ann and Bob playing a bet on an uncertain event $A$ (such as whether their favorite team wins an upcoming football match), which is depicted on Figure 1. Ann puts $x$ dollars and Bob puts $y$ dollars in the

[2]See Lewandowski and Woźny (2022) for some recent discussion on selling vs. short-selling prices.

pot. If $A$ occurs, Ann wins the whole pot. Otherwise, Bob gets it. Ann's net profit is therefore $y$ if $A$ occurs, and $-x$ otherwise. Since Bob's net profit is the exact opposite, for a given probability of $A$, at most one side of the bet may have positive expected value. So if Ann and Bob care about expected profit and they both prefer to bet, their beliefs must differ, i.e. Ann' estimate of the likelihood of $A$ is sufficiently higher than Bob's estimate. However, some people may like to play despite the negative expected value, and hence, for some bet they may prefer to accept either side of it rather than not bet at all. We call such people uncertainty lovers. Most people exhibit the opposite behavior, which we will call *uncertainty aversion*: they reject at least one side of any bet.

There are at least two reasons why a betting position is rejected. Either the agent knows he dislikes it or he is unsure, and, out of caution, prefers not to bet. We call an agent *surely uncertainty averse* if he strictly dislikes at least one side of any bet. The residual component, whereas the agent cannot make up his mind, will be referred to as preference imprecision. Sure uncertainty aversion is closely related to the idea that losses loom larger than gains. Recall that the net profits of two sides of the same bet are the exact opposites. Consider the bet in 1 where $y = x$ and the event is symmetric, meaning that swapping the sides of the bet does not alter its attractiveness. Then, if one side of the bet is disliked, which is the case if the agent is surely uncertainty averse, then so is the other side. This implies the standard definition of loss aversion for risk, according to which people dislike equal chance gambles of winning or losing the same monetary amount (Kahneman and Tversky, 1979).

In order to numerically capture uncertainty aversion and sure uncertainty aversion, we will use the concepts of a short-selling price (or willingness to accept, WTA) and buying price (or willingness to pay, WTP) and their extensions suggested by Eisenberger and Weber (1995); Cubitt et al. (2015). Placing bets can be understood as the transaction of issuing and purchasing a lottery (prospect) ticket. In the example given above, Bob issues a ticket to Ann that pays $x + y$ if $A$ occurs and 0 otherwise. The price for this ticket is $x$. Since Ann buys the ticket (she accepts a bet), it means that $x$ does not exceed the highest price she is willing to pay for it (WTP). Bob, on the other hand, agrees to issue the ticket for the price $x$ which means that $x$ is at least as large as the lowest price he is ready to accept in exchange for the prospect (WTA).

Under complete preferences, WTP and WTA are indifference prices, at which Ann (Bob, respectively) is indifferent between buying (issuing) and not buying (not issuing) the ticket. This might not be the case if preferences are incomplete, as then we must distinguish between indecision and indifference. Thus, besides the buying (respectively, short-selling) price, we define the so called no-buying (no-short-selling) price that is the smallest price (the greatest) at which the agent is confident that the status quo is better than buying (short-selling).

**Contribution** Our contribution is as follows. We define uncertainty aversion (UA) as a new behavioral condition. Our definition differs from many standard definitions of ambiguity/uncertainty aversion in several respects. First, it uses certainty as the benchmark for neutrality to uncertainty rather than subjective expected utility or probabilistically sophisticated preferences (Ghirardato and Marinacci, 2002; Epstein, 2004) and, consequently, a different notion of comparative notion of uncertainty aversion. To put it simply, our notion treats uncertainty the same as risk and compares both to certainty, whereas many standard definitions treat uncertainty as something on top of risk and compare it to preferences under risk. We distinguish the part of UA that the agent is certain about, and the remaining part due to preference imprecision. We extend the standard definition of loss aversion/loss-loving of Kahneman and Tversky (1979) from risk and complete preferences to ambiguity and incomplete preferences. Under mild assumptions, we show the equivalence between not-loss loving and UA as well as loss aversion and the sure part of UA.

We show how to measure UA, the sure part of UA and preference imprecision using differences in counterparts of indifference prices for incomplete preferences. Unlike many standard measures of ambiguity aversion, which measure the size of the set of subjective beliefs and, hence, are unobservable, our measures are monetary and are interpreted as uncertainty premiums. Our setup is that of Savage (1954) and we impose only minimal assumptions which allow to correctly identify these prices. This defines a very broad class of models, allowing for loss aversion, preference incompleteness, and no separation of belief and taste. Nevertheless, we use the Multi-Utility Multi-Prior (MUMP) model as illustration.

We prove that UA is equivalent to WTA being greater than WTP. We also define its comparative version ("more uncertainty averse" agent) to show that the WTA-WTP disparity is a cardinal measure of UA. We do the same for the sure part of UA and its corresponding measure involving no-buying and no-short-selling prices.

We finally show how to decompose the WTA-WTP disparity using these measures, that is one attributed to sure UA (=loss aversion) and the other to preference imprecision. This decomposition allows to disentangle the two channels driving the WTA-WTP disparity. We report results of an experiment we conducted to illustrate how our approach can be used to decide which explanation drives the disparity to a greater extent. We allow people in the elicitation task to express at which prices they are unsure which option, buying (short-selling) or the status quo, is better. For this purpose we adopt the modified multiple price list (MPL) procedure[3] proposed by Cubitt et al. (2015) and also compatible with Agranov and Ortoleva, 2023.

# 2 The model

## 2.1 Preliminaries

Let $S$ be a set of states. Subsets of the state space are called events. The outcome set is $\mathbb{R}$, with real numbers designating income amounts, and zero denoting the status quo. A prospect is a mapping from $S$ to $\mathbb{R}$ such that $f(S)$ has finitely many elements. The set of all prospects is denoted by $\mathcal{F}$. An important special case arises if every measurable subset of $S$ is assigned a probability. Risk is a special case where $(S, \mathcal{S}, \Pi)$ is a probability space, and if the induced probability distributions of two prospects coincide, then the prospects are preferentially equivalent. A prospect $f$ such that $f(s) = \lambda$ for all $s \in S$, where $\lambda \in \mathbb{R}$, is called a constant prospect and denoted by $\lambda$. A nonzero prospect is a prospect different than prospect 0. Prospects $f, g$ are comonotonic if for all $s, t \in S$, $f(s) > f(t)$ implies $g(s) \geq g(t)$. We say that a pair of prospects $f, g$ is a perfect hedge if $f + g = \theta$ for some $\theta \in \mathbb{R}$. We will write $f \geq g$ if $f(s) \geq g(s)$ for all $s \in S$, $f > g$ if $f(s) > g(s)$ for all $s \in S$. For a prospect $f$ we also define $\min f := \min_{s \in S} f(s)$ and $\max f := \max_{s \in S} f(s)$. A binary prospect $f$ will be denoted by $(x, y; A)$, with an event $A$ and payoffs $x, y$, where $f(A) = \{x\}$ and $f(A^c) = \{y\}$.

Let $\succcurlyeq$ be a binary relation on $\mathcal{F}$. For $f, g \in \mathcal{F}$, we say that $f$ and $g$ are comparable if $f \succcurlyeq g$ or $g \succcurlyeq f$. Otherwise, they are incomparable, which we write as $f \bowtie g$. $\succcurlyeq$ is complete if and only if all pairs are comparable. Given $\succcurlyeq$ we define $\sim$, $\succ$ in the usual way, i.e. $f \sim g \iff f \succcurlyeq g \ \wedge \ g \succcurlyeq f$ and $f \succ g \iff f \succcurlyeq g \ \wedge \ g \not\succcurlyeq f$. Whenever $f \succcurlyeq 0$, we will say the decision maker accepts $f$. Otherwise, when $f \not\succcurlyeq 0$, we will say that she rejects $f$. If $0 \succ f$, we shall say that the decision maker dislikes $f$. If preferences are complete then for any two prospects $f, g$, $f \not\succcurlyeq g$ is equivalent to $g \succ f$. However, under incomplete preferences $f \not\succcurlyeq g$ is compatible both with $g \succ f$ as well as $g \bowtie f$. This corresponds to two reasons why the decision maker rejects $f$ in the choice between $f$ and $g$, either because she thinks $g$ is surely better than $f$ or because $f$ and $g$ are incomparable. We impose the following axioms on $\succcurlyeq$.

**B0 (Preorder)**: $\succcurlyeq$ is reflexive and transitive.

[3] The standard MPL procedure is described in Andersen et al. (2006).

**B1 (Monotonicity)**: If $f \geq g$ then $f \succcurlyeq g$. If, in addition, $f \neq g$, then $f \succ g$.

**B2 (Continuity)**: The following sets are closed (with respect to the Euclidean topology on $\mathbb{R}^n$) for all $f \in \mathcal{F}$:

$$\mathrm{nW}(0) := \{f \in \mathcal{F} : f \not\preccurlyeq 0\}$$
$$\mathrm{nB}(0) := \{f \in \mathcal{F} : 0 \not\preccurlyeq f\}$$

## 2.2 Boundary prices and their basic properties

For a prospect $f$, we define the following four price functionals:

$$\text{buying price} \quad B : \mathcal{F} \to \mathbb{R} \qquad B(f) = \max\{\theta \in \mathbb{R} : f - \theta \succcurlyeq 0\}, \tag{1}$$
$$\text{no buying price} \quad B_n : \mathcal{F} \to \mathbb{R} \qquad B_n(f) = \min\{\theta \in \mathbb{R} : 0 \not\preccurlyeq f - \theta\}, \tag{2}$$
$$\text{selling-short price} \quad B^* : \mathcal{F} \to \mathbb{R} \qquad B^*(f) = \min\{\theta \in \mathbb{R} : \theta - f \succcurlyeq 0\}, \tag{3}$$
$$\text{no selling-short price} \quad B_n^* : \mathcal{F} \to \mathbb{R} \qquad B_n^*(f) = \max\{\theta \in \mathbb{R} : 0 \not\preccurlyeq \theta - f\}, \tag{4}$$

Figure 2 depicts the four prices defined above for a binary prospect $(x, y; A)$. We state some basic properties of the prices. All proofs are in the Appendix.

**Proposition 1.** *For $X \in \{B, B_n, B^*, B_n^*\}$ and every prospect $f$, a unique $X(f)$ exists and satisfies the mean property, i.e.*

$$\min f \leq X(f) \leq \max f.$$

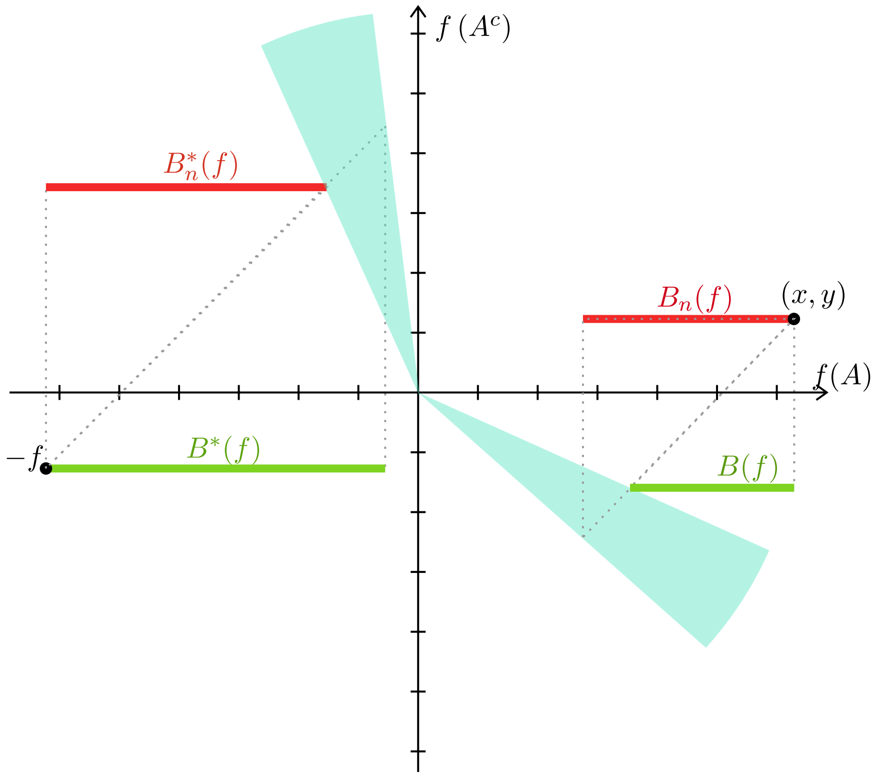
*The prices satisfy*

$$B_n(f) \geq B(f), \quad B^*(f) \geq B_n^*(f). \tag{5}$$

*Moreover, if there is prospect $f$ such that at least one of the inequalities in* (5) *is strict, then preferences are incomplete.*

The following relationships between $B^*, B$ and $B_n^*, B_n$ hold (see Lewandowski and Woźny (2022) for earlier results for $B$ and $B*$).

Figure 2: The boundary prices for a binary prospect $(x, y; A)$




**Proposition 2.** *For all prospects $f$ and all scalars $\theta$ the following holds:*

$$B^*(f) + B(\theta - f) = \theta \tag{6}$$

$$B_n^*(f) + B_n(\theta - f) = \theta \tag{7}$$

### 2.3 Uncertainty aversion, the WTA-WTP gap and its decompositions

We define uncertainty aversion and show that it captures the WTA-WTP disparity.

**Definition 1.** $\succcurlyeq$ *is **uncertainty averse** if $f \succcurlyeq 0$ implies $-f \not\succcurlyeq 0$ for all $f \in \mathcal{F} \setminus \{0\}$.*

**Proposition 3.** *The following are equivalent:*

*(i)* $\succcurlyeq$ *is uncertainty averse.*

*(ii)* $B^*(f) - B(f) > 0$ *for every* $0 \neq f \in \mathcal{F}$.

Incompleteness is a global notion. We define a local notion of preference imprecision (PI) that holds for a given prospect $f$.

**Definition 2.** *A decision maker is imprecise for prospect $f$ if there exists $\theta \in \mathbb{R}$ such that $f + \theta \bowtie 0$. Otherwise, he is precise for prospect $f$.*

**Proposition 4.** *$\succcurlyeq$ is imprecise for prospect $f$ if and only if $B_n(f) > B(f)$ and precise if and only if $B_n(f) = B(f)$. Similarly, $\succcurlyeq$ is imprecise for prospect $-f$ if and only if $B^*(f) > B_n^*(f)$ and precise if and only if $B^*(f) = B_n^*(f)$.*

In order to filter out the part of UA that the agent is confident about, we need to clean it from the imprecision part. If we think of the gambling situation both from the perspective of a buyer and short-seller, we need to clean UA both from imprecision for $f$ and imprecision for $-f$. This leads to the notion of sure UA.

**Definition 3.** *$\succcurlyeq$ is **surely uncertainty averse** if $0 \not\succ f$ implies $0 \succ -f$ for all $f \in \mathcal{F} \setminus \{0\}$.*

**Proposition 5.** *The following are equivalent:*

*(i) $\succcurlyeq$ is surely uncertainty averse.*

*(ii) $B^*(f) - B(f) > 0$ and $B_n^*(f) - B_n(f) \geq 0$ for every $0 \neq f \in \mathcal{F}$.*

Consider a uncertainty averse agent and some prospect $f$. By Proposition 5, we have $B^*(f) > B(f)$. By definition of $B^*$ and $B$, we know that for all $\theta$ in between $B(f)$ and $B^*(f)$, the agent will neither accept $f - \theta$ nor $\theta - f$. We partition this set to capture two motives (due to indecision or confidence) why the agent rejects either one of the two betting positions.

$$\text{PI} := \{\theta \in (B(f), B^*(f)) :\ 0 \bowtie f - \theta \ \vee \ 0 \bowtie \theta - f\},$$
$$\text{sure UA} := \{\theta \in (B(f), B^*(f)) :\ 0 \succcurlyeq f - \theta \ \wedge \ 0 \succcurlyeq \theta - f\}.$$

By (1)–(4), the size of these sets can measured by the respective boundary prices which leads to:

$$\text{decomposition 1:} \quad \underbrace{B^*(f) - B(f)}_{\text{UA}} = \underbrace{B^*(f) - B_n^*(f)}_{\text{PI for } -f} + \underbrace{B_n^*(f) - B_n(f)}_{\text{sure UA}} + \underbrace{B_n(f) - B(f)}_{\text{PI for} f} \qquad (8)$$

We can understand gambling as embedded in one side of the betting situation. In this case, we define the following notion of uncertainty aversion that lies half-way between uncertainty aversion and sure uncertainty aversion.

**Definition 4.** *$\succcurlyeq$ is **strongly uncertainty averse** if $f \succcurlyeq 0$ implies $0 \succ -f$ for all $f \in \mathcal{F} \setminus \{0\}$.*

**Proposition 6.** *The following are equivalent:*

*(i) The decision maker is strongly uncertainty averse.*

*(ii)* $B^*(f) - B(f) > 0$ *and* $B^*(f) - B_n(f) \geq 0$ *and* $B_n^*(f) - B(f) \geq 0$ *for every* $0 \neq f \in \mathcal{F}$.

This new definition leads to the second way we may partition the interval $(B(f), B^*(f))$ for a uncertainty averse individual. Since we have two betting positions, we define two partitions, one for each betting position.

$$\begin{aligned}
\mathrm{PI}_f &:= \{\theta \in (B(f), B^*(f)) : \; 0 \bowtie f - \theta\} \\
\text{strong UA}_f &:= \{\theta \in (B(f), B^*(f)) : \; 0 \succcurlyeq f - \theta\} \\
\mathrm{PI}_{-f} &:= \{\theta \in (B(f), B^*(f)) : \; 0 \bowtie \theta - f\} \\
\text{strong UA}_{-f} &:= \{\theta \in (B(f), B^*(f)) : \; 0 \succcurlyeq \theta - f\}.
\end{aligned}$$

This leads to the following two decompositions.

$$\text{decomposition 2a:} \qquad \underbrace{B^*(f) - B(f)}_{\text{UA}} = \underbrace{B^*(f) - B_n(f)}_{\text{strong UA}_f} + \underbrace{B_n(f) - B(f)}_{\text{PI}_f} \tag{9}$$

$$\text{decomposition 2b:} \qquad = \underbrace{B^*(f) - B_n^*(f)}_{\text{PI}_{-f}} + \underbrace{B_n^*(f) - B(f)}_{\text{strong UA}_{-f}} \tag{10}$$

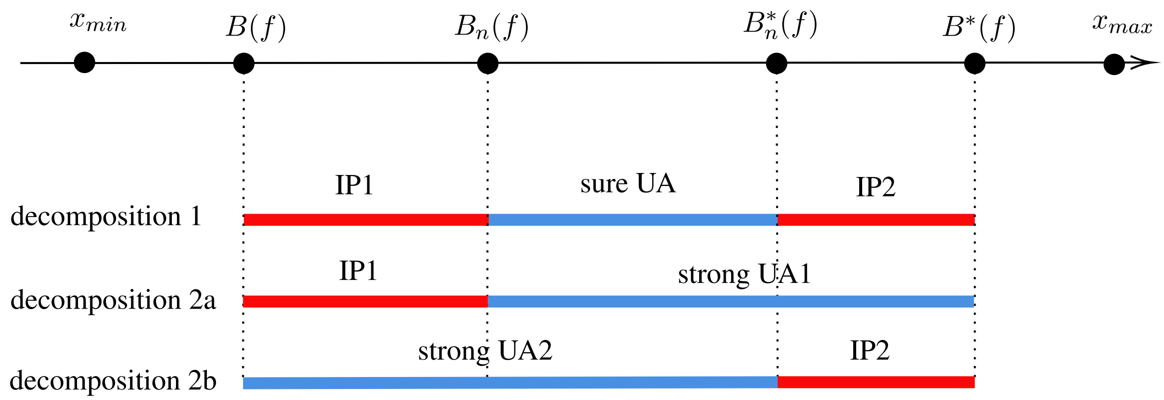
Figure 3 depicts the possible decompositions. Note that sure UA implies strong UA and strong UA implies UA – this can be inferred directly, or through the above propositions that also characterize these three notions in terms of boundary prices.

## 2.4 Binary symmetric prospects.

An important class of prospects are symmetric binary prospects that are the counterparts of equal-chance binary gambles for risk. Following Ramsey (Parmigiani and Inoue, 2009, p.78)[4], we assume that there exists a so called $\frac{1}{2}$-*probability event*, i.e. a subset $A$ of $S$ such that $(x, y; A) \sim (y, x; A)$ for every $x, y \in X$. Prospects of the form $(x, y; A)$ where $A$ is a $\frac{1}{2}$ probability event will be called symmetric binary prospects. For such prospects, the preference imprecision part for prospect $f$ and $-f$ is the same.

[4]See also (Gul, 1992, Assumption 3).

Figure 3: Uncertainty aversion (captured by the difference between the short-selling price and the buying price of a prospect decomposed into preference imprecision (blue) and sure uncertainty aversion (red).




**Proposition 7.** *For $f = (x, y; A)$ where $x, y \in \mathbb{R}$ and $A$ is a $\frac{1}{2}$-probability event, the following holds*

$$B_n(f) - B(f) = B^*(f) - B_n^*(f).$$

This and other symmetry features of binary symmetric prospects make them attractive in applications. We demonstrate it in Proposition 9 as well as in section 3.2 in connection to the standard notion of loss aversion. We also use them in our experiment. The implications of Proposition 7 are graphically demonstrated in Figure 4.

# 3 Discussion

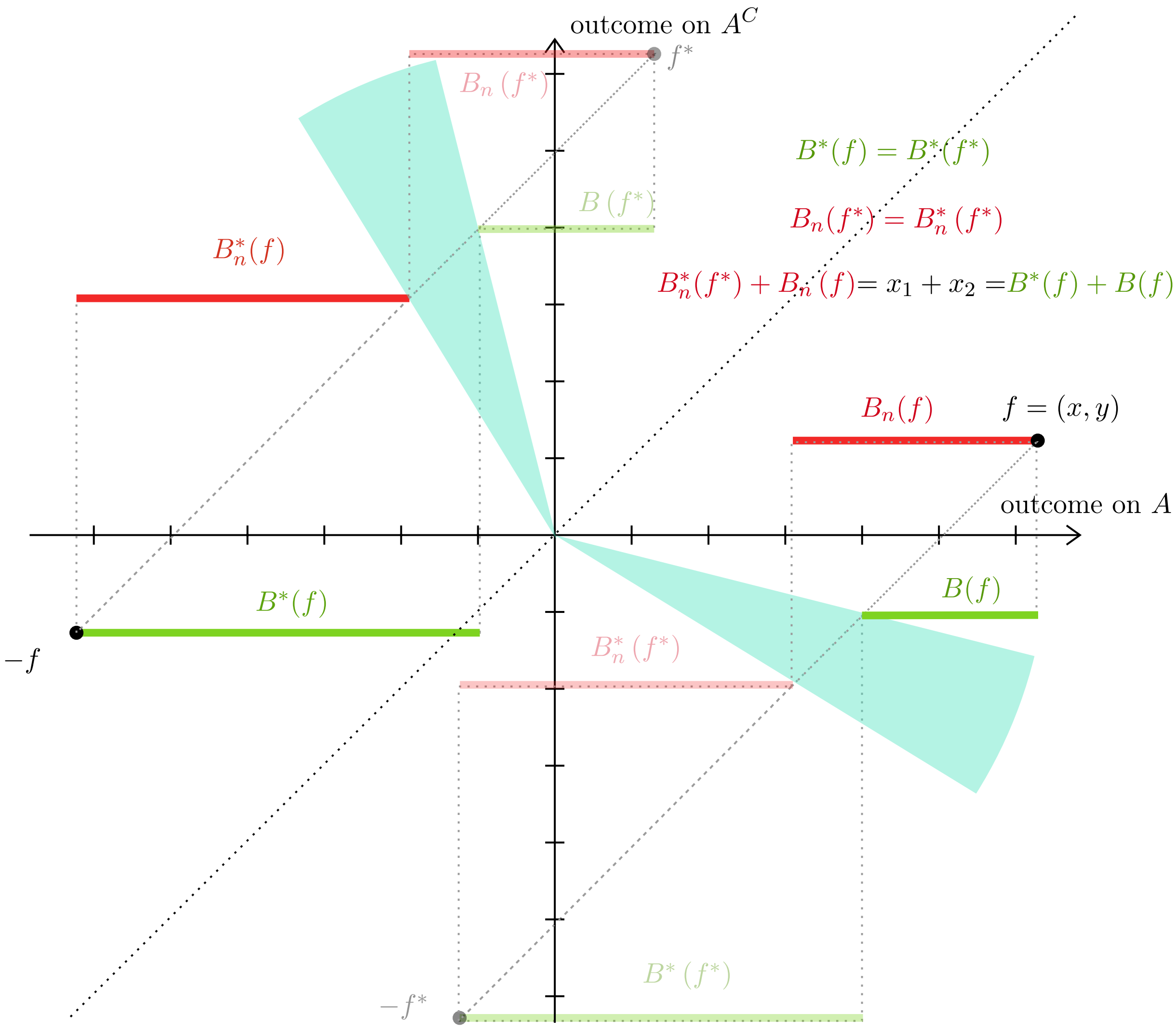
## 3.1 WTA-WTP disparity in the Multi-Utility Multi-Prior model

We illustrate our results using the multi-utility multi-prior (MUMP) model (Galaabaatar and Karni, 2013; Hara and Riella, 2023; Borie, 2023). MUMP is more specific than our model, yet general enough to capture preference imprecision and loss aversion at the same time. We follow Hara and Riella (2023) and assume that the outcome set is $X = [a, b]$ for some $a, b \in \mathbb{R}$ with $a < 0 < b$.[5] MUMP has originally been stated in the Anscombe et al. (1963) analytical setup with acts being defined as $\Delta(X)^{S'}$, where $\Delta(X)$ is the set of probability measures on $X$ and $S'$ is a finite set of states. We formulate MUMP in the Savage (1954) setup adopted in this paper by focusing on constant AA-acts and restricting the set of states to be finite.[6]

[5]Other characterizations of MUMP assume finite outcome space, which is not suitable for modeling boundary prices. We assume throughout this section that all the discussed properties hold on $X$ rather than on $\mathbb{R}$.

[6]Note, however, that we still assume the existence of a $\frac{1}{2}$-probability event.

Figure 4: The boundary prices for a binary symmetric prospect



**Definition 5.** *$\succcurlyeq$ on $\mathcal{F}$ has a MUMP representation if there exist a compact set $\mathcal{U}$ of continuous strictly increasing real-maps on $X$ and a compact set $\Pi^u$, $u \in \mathcal{U}$, of probability measures on $S$ such that for each $f, g \in \mathcal{F}$,*

$$f \succcurlyeq g \iff \int_S u(f)d\mu \geq \int_S u(g)d\mu \quad \text{for every } (\mu, u) \in \Phi. \tag{11}$$

*where $\Phi = \{(\mu, u) : \ u \in \mathcal{U}, \ \mu \in \Pi^u\}$.*

Note that the set of utilities and priors may depend on each other. An important special case is when they are independent. In this case, instead of a separate set of priors $\Pi^u$ for each utility $u \in \mathcal{U}$ there is a single set of priors $\Pi$.

For $(\mu, u) \in \Phi$ we define $B_{\mu,u} : \mathcal{F} \to \mathbb{R}$ and $B^*_{\mu,u} : \mathcal{F} \to \mathbb{R}$ as

$$\sum_{s \in S} \mu(s) u(f(s) - B_{\mu,u}(f)) = 0, \tag{12}$$

$$\sum_{s \in S} \mu(s) u(B^*_{\mu,u}(f) - f(s)) = 0. \tag{13}$$

**Proposition 8.** *If $\succcurlyeq$ has a MUMP representation where the set of priors and utilities is $\Phi$ then for any $f \in \mathcal{F}$, we have*

$$B(f) = \min_{(\mu,u)\in\Phi} B_{\mu,u}(f),$$

$$B_n(f) = \max_{(\mu,u)\in\Phi} B_{\mu,u}(f),$$

$$B^*_n(f) = \min_{(\mu,u)\in\Phi} B^*_{\mu,u}(f),$$

$$B^*(f) = \max_{(\mu,u)\in\Phi} B^*_{\mu,u}(f).$$

The following property that holds for symmetric binary prospects under MUMP states that the sure UA part of the decomposition coincides with the minimum over $(\mu, u) \in \Phi$ of the $B^*_{\mu,u} - B_{\mu,u}$.

**Proposition 9.** *Assume $\succcurlyeq$ has a MUMP representation with the set of utilities and priors $\Phi$. For $f = (x, y; A)$ where $x, y \in \mathbb{R}$ and $A$ is a $\frac{1}{2}$-probability event, the following holds*

$$B^*_n(f) - B_n(f) = \min_{(\mu,u)\in\Phi} \left(B^*_{\mu,u}(f) - B_{\mu,u}(f)\right). \tag{14}$$

We now give a numerical example based on (Cerreia-Vioglio et al., 2015, Appendix D). For $\alpha, \lambda \in \mathbb{R}_{++}$, we assume $u_{\alpha,\lambda}$ be a utility function given by:

$$u_{\alpha,\lambda}(x) = \begin{cases} x^{\alpha} & \text{for } x \geq 0, \\ -\lambda(-x)^{\alpha} & \text{for } x < 0. \end{cases}$$

We assume $\succcurlyeq$ has a MUMP representation with $\mathcal{U} = \{u_{0.5,2}, u_{0.25,4}\}$ being the set of utilities and $\Pi^u$, the set of priors for $u \in \mathcal{U}$. We consider a binary symmetric prospect $f = (x, 0; A)$, where $x \in X$, $x > 0$ and $A$ is a $\frac{1}{2}$-probability event. Since $\mu(A) = 0.5$ for all $\mu \in \Pi^u$, $u \in \mathcal{U}$, we will omit $\mu$ and focus on $u$. We have

$$B_u(f) = \frac{1}{1 + \lambda^{\frac{1}{\alpha}}} x,$$
$$B^*_u(f) = \frac{1}{1 + \lambda^{-\frac{1}{\alpha}}} x.$$

In view of Proposition 8, we have

$$B(f) = B_{u_{0.25,4}}(f) = 0.3891$$
$$B_n(f) = B_{u_{0.5,2}}(f) = 20$$
$$B^*_n(f) = B^*_{u_{0.5,2}}(f) = 80$$
$$B^*(f) = B^*_{u_{0.25,4}}(f) = 99.611$$

### 3.2 Uncertainty aversion versus loss aversion

Our definition of uncertainty aversion asks the decision maker to put oneself in each of the two opposing positions, either two sides of a bet or a position of a buyer and issuer of a lottery ticket. This is naturally related to the treatment of gains and losses. We wish to establish formal relationship between uncertainty aversion and loss aversion.

The standard definition of loss aversion for risk (Kahneman and Tversky, 1979) is that people dislike equal chance gambles of winning or losing the same nonzero amount. In this section we extend this definition to uncertainty and incomplete preferences. We replace equal chance gambles with symmetric binary prospects. We also clarify what it means to “dislike” a prospect. Following our previous discussion, the decision maker may either be convinced that he prefers the status quo rather than accepting a prospect, or he may not be convinced and thus, by inertia, may refuse to accept the prospect. This leads to the following two definitions.

**Definition 6.** *$\succcurlyeq$ is **loss averse** (LA) if $0 \succ (x, -x; A)$ holds for every $x \in \mathbb{R} \setminus \{0\}$ and any $\frac{1}{2}$-probability event $A$.*

**Definition 7.** *$\succcurlyeq$ is **not loss-loving** (not-LL) if $(x, -x; A) \not\succ 0$ for every $x \in \mathbb{R} \setminus \{0\}$ and any $\frac{1}{2}$-probability event $A$.*

We establish the following relationships between uncertainty aversion and loss aversion.

**Proposition 10.** *The following hold*

*(i) If $\succcurlyeq$ is uncertainty averse then it is not loss-loving.*

*(ii) If $\succcurlyeq$ is surely uncertainty averse then it is loss averse.*

For the converse implications we need more structure.

**Lemma 1.** *Assume that $\succcurlyeq$ has a MUMP representation with $\Phi = \{(\mu, u) : \ u \in \mathcal{U}, \ \mu \in \Pi^u\}$ being some set of prior-utility pairs satisfying the conditions of Definition 5.*

*(i) $A$ is a $\frac{1}{2}$-probability event iff $\mu(A) = \frac{1}{2}$ for all $\mu \in \Pi^u$ and all $u \in \mathcal{U}$.*

*(ii) Loss aversion holds iff $u(x) \leq -u(-x)$, $\forall x \in X$ holds for every $u \in \mathcal{U}$, and $u(x) < -u(-x)$, $\forall x \in X \setminus \{0\}$ holds for some $u \in \mathcal{U}$.*

*(iii) Not loss loving holds iff $u(x) < -u(-x)$, $\forall x \in X \setminus \{0\}$ holds for some $u \in \mathcal{U}$.*

**Proposition 11.** *Assume that $\succcurlyeq$ has a MUMP representation.*

*(i) If $\succcurlyeq$ is not loss-loving then it is uncertainty averse.*

*(ii) If $\succcurlyeq$ is loss-averse then it is strongly uncertainty averse.*

### 3.3 Uncertainty aversion vs. uncertainty/ambiguity aversion vs. risk aversion

Cite Epstein (1999) RES paper Definition of UA and also Gilboa, Marinacci (2011) survey where they say that the UA has two components: a comparative notion of UA (their notion) and a benchmark for neutrality to ambiguity (SEU preferences). In our case a comparative notion is given in the next subsection and the benchmark for neutrality to uncertainty is certainty (expained in this subsection). We can also mention vague preferences and incommensurability discussion in the philosophy literature.

We now discuss the interpretation of the UA disparity $B^*(f) - B(f)$ for a prospect $f$. The disparity can be seen as an extension of the notion of risk premium to uncertainty. Risk aversion is measured by pricing risk vs. certainty.

$$f = \underbrace{\mathrm{EV}(f)}_{\text{riskless part}} + \underbrace{f - \mathrm{EV}(f)}_{\text{risk}}.$$

$$\text{risk premium}(f) = \underbrace{\mathrm{EV}(f)}_{\text{value without risk}} - \underbrace{\mathrm{CE}(f)}_{\text{value with risk}}.$$

Uncertainty or ambiguity aversion is measured (e.g. by the size of the belief set) on the scale between risk (one PDF) and complete ignorance (all PDFs). Our measure of uncertainty aversion **compares an uncertainty to certainty**. Instead of EV only available for risk, we use the idea of a **perfect hedge**. Recall that a pair of prospects $f, f^*$ is a perfect hedge if $f^* = \theta - f$ for some $\theta \in \mathbb{R}$. Thus, adding a perfect hedge to a prospect, eliminates uncertainty completely. For $f \in \mathcal{F}$ and $\theta \in \mathbb{R}$ let $\theta_1, \theta_2 \in \mathbb{R}$ be the uncertainty-free values of $f$ and $\theta - f$, respectively. Under risk, $\theta_1 = \mathrm{EV}(f)$, $\theta_2 = \mathrm{EV}(\theta - f)$ so they satisfy

$$\theta_1 + \theta_2 = \theta. \tag{15}$$

For other sources of uncertainty, $\theta_1, \theta_2$ may be unknown, but we assume that (15) holds. Then

$$B^*(f) - B(f) \overset{\text{by Prop}}{=} \theta - B(\theta - f) - B(f) \overset{\text{by }(*)}{=} \underbrace{\theta_1 - B(f)}_{\text{uncertainty premium for } f} + \underbrace{\theta_2 - B(\theta - f)}_{\text{uncertainty premium for } \theta - f}$$

Thus, the UA disparity is the minimal sum of compensation amounts, which, after being split optimally and added to each side of the bet, will make them both acceptable.

### 3.4 More uncertainty averse individual

We denote by $\succcurlyeq_i$, a preference relation of agent $i$. Similarly, we denote by $B_i, B_i^*, B_{ni}, B_{ni}^*$ the buying, short-selling, no-buying and no-short-selling price of agent $i$, respectively.

**Definition 8.** *We say that* $\succcurlyeq_1$ *is* ***more UA*** *than* $\succcurlyeq_2$ *if for every* $0 \neq f \in \mathcal{F}$ *it holds:*

$$f \succcurlyeq_1 0 \quad and \quad \epsilon - f \succcurlyeq_1 0 \quad for\ some\ \epsilon \in \mathbb{R} \quad \Rightarrow \quad \exists \delta \in \mathbb{R}: \quad f - \delta \succcurlyeq_2 0 \quad and \quad \delta + \epsilon - f \succcurlyeq_2 0.$$

**Proposition 12.** $\succcurlyeq_1$ *is more UA than* $\succcurlyeq_2$ *if and only if*

$$B_1^*(f) - B_1(f) \geq B_2^*(f) - B_2(f) \quad for\ every\ 0 \neq f \in \mathcal{F}. \tag{16}$$

**Definition 9.** *We say that* $\succcurlyeq_1$ *is* ***more surely UA*** *than* $\succcurlyeq_2$ *if for every* $0 \neq f \in \mathcal{F}$ *it holds:*

$$0 \succcurlyeq_2 f \quad and \quad 0 \succcurlyeq_2 \epsilon - f \quad for\ some\ \epsilon \in \mathbb{R} \quad \Rightarrow \quad \exists \delta \in \mathbb{R}: \quad 0 \succcurlyeq_1 f - \delta \quad and \quad 0 \succcurlyeq_1 \delta + \epsilon - f.$$

**Proposition 13.** $\succcurlyeq_1$ *is more surely UA than* $\succcurlyeq_2$ *if and only if*

$$B_{n1}^*(f) - B_{n1}(f) \geq B_{n2}^*(f) - B_{n2}(f) \quad for\ every\ 0 \neq f \in \mathcal{F}. \tag{17}$$

### 3.5 More uncertain prospects

Given two prospects $f, g$, we say that $g$ is more uncertain than $f$ if $g - f$ is a nonconstant prospect comonotonic to $f$. If $g$ is more uncertain than $f$ and $g - f \not\succcurlyeq 0$, we say that $f$ uncertainty-dominates $g$. If $g$ is more uncertain than $f$ and $g - f \not\succ 0$, we say that $f$ strongly uncertainty-dominates $g$. We say that $\succcurlyeq$ is monotonic with respect to uncertainty-dominance if $f \succcurlyeq g$ whenever $f$ uncertainty-dominates $g$. We say that $\succcurlyeq$ is monotonic with respect to strong uncertainty-dominance if $f \succcurlyeq g$ whenever $f$ strongly uncertainty-dominates $g$.

**Proposition 14.** *The statement: If $g$ is more uncertain than $f$ then*

$$B^*(f) - B(f) \leq B^*(g) - B(g) \tag{18}$$

$$and \quad B_n^*(f) - B_n(f) \leq B_n^*(g) - B_n(g), \tag{19}$$

*is implied by each of the following two sets of conditions:*

*(i)* $\succcurlyeq$ *satisfies sure uncertainty aversion and is monotonic with respect to uncertainty-dominance,*

*(ii)* $\succcurlyeq$ *satisfies uncertainty aversion and is monotonic with respect to strong uncertainty-dominance.*

Remark 1: Note that uncertainty dominance neither implies $B(f) \geq B(g)$ nor $B^*(f) \leq B^*(g)$. Give example. Remark 2: Neither $B$ nor $B^*$ represents preferences. If we assumed comonotonic additivity, then we could have some preferences $f$ vs $g$ for $f$ uncertainty-dominating $g$ even without monotonicity wrt to uncertainty-dominance.

**Stronger version of loss aversion** A stronger version of loss aversion is that $0 \succ (x, -x; A) \succcurlyeq (y, -y; A)$ for all $y > x > 0$ and all $\frac{1}{2}$-probability event $A$. Note that it is implied by LA and monotonicity with respect to uncertainty-dominance. Indeed, take an arbitrary $\frac{1}{2}$-probability event $A$, an arbitrary $x, y \in \mathbb{R}$ such that $y > x > 0$ and define $\epsilon := y - x$. Because $\epsilon > 0$ so $(x, -x; A)$ is comonotonic to $(\epsilon, -\epsilon; A)$. Furthermore, by LA, $(\epsilon, -\epsilon; A) \prec 0$, and hence $(\epsilon, -\epsilon; A) \not\succcurlyeq 0$. Thus, $(x, -x; A)$ uncertainty-dominates $(y, -y; A) = (x, -x; A) + (\epsilon, -\epsilon; A)$. By monotonicity with respect to uncertainty-dominance $(x, -x; A) \succcurlyeq (y, -y; A)$.

### 3.6 The Ellsberg preferences

Note that uncertainty dominance captures hedging and more variability in outcomes. However, it does not address source preferences. In order to compare gambles that are bets on different sources, we introduce the following property. A source is formally an algebra of events. To make things simpler, we will however, focus on the binary partitions of the state space $(E, E^c)$ where $E$ is a nontrivial subset of $S$ (different than $S$ and $\emptyset$) and $E^c = S \setminus E$. We say that source $(E, E^c)$ dominates another source $(F, F^c)$ if the following holds for all $x_1 > \max(0, x_2)$ and $x'_1 > \max(0, x'_2)$:

$$(x_1, x_2; E) \succ (x_1, x_2; F)$$
$$(x'_1, x'_2; E^c) \succ (x'_1, x'_2; F^c)$$

**Proposition 15.** *If source $(E, E^c)$ dominates source $(F, F^c)$, then $B^*(x, y; E) - B(x, y; E) \geq B^*(x, y; F) - B(x, y; F)$.*

*Proof.* Let $B_1 := B(x, y; F)$ and $B_1^* := B^*(x, y; F)$. If $(E, E^c)$ dominates $(F, F^c)$, then $(x - B_1, y - B_1; E) \succ (x - B_1, y - B_1; F) \succcurlyeq 0$, where the last inequality follows from the definition of $B_1$. By transitivity, $(x - B_1, y - B_1; E) \succ 0$. Hence, by the definition of $B$ and $B_n$, we have $B(x, y; E) \geq B_1 = B(x, y; F)$ and $B_n(x, y; E) > B_1 = B(x, y; F)$. Similarly, $(B_1^* - y, B_1^* - x; E^c) \succ (B_1^* - y, B_1^* - x; F^c) = (B_1^* - x, B_1^* - y; F) \succcurlyeq 0$, where the last inequality follows from the definition of $B_1^*$. By transitivity, $(B_1^* - x, B_1^* - y; E) = (B_1^* - y, B_1^* - x; E^c) \succ 0$. Hence, by the definition of $B^*$ and $B_n^*$, we have $B^*(x,y;E) \leq B_1^* = B^*(x,y;F)$ and $B_n^*(x,y;E) < B_1^* = B^*(x,y;F)$. So we obtain $B^*(x,y;E) - B(x,y;E) \leq B^*(x,y;F) - B(x,y;F)$ and $B_n^*(x,y;E) - B_n(x,y;E) < B^*(x,y;F) - B(x,y;F)$. $\square$

# 4 Experimental illustration

## 4.1 Method

We elicited buying, no-buying, short-selling, no short-selling prices for the risky and ambiguous gambles using a multiple price list with three columns. Our MPLs are lists of ternary choices between a status quo option on the right, for example: I certainly would not buy, a varying option on the left, for example: I certainly would buy for the price of..., and "I am not sure option" in the middle. The left-hand option changes monotonically (the price for a gamble increases). A rational subject should buy if the price is low and not buy if the price is high. She may not be sure at some range of intermediate prices. The row on which a participant switches the left-hand side to either the middle or the right-hand side identifies a range of possible values for the buying price. The row on which a participant switches to the right-hand side from either the left-hand or the middle side identifies a range of possible values for the no-buying price. We use the midpoint of these ranges in our analysis to identify the buying and no buying (similarly short-selling and no short-selling prices), but the results are similar if we use the minimum or maximum value.

The short-selling and no short-selling prices are elicited in a similar way, except for now for the rational subject the switching occurs in the opposite direction because in this case the subject receives the sure money in return for issuing a lottery ticket ((instead of paying it to buy a ticket). Hence a rational subject should short-sell if the price is high and not short-sell if the price is low. Participants received extensive training on MPLs, and correctly answered several comprehension questions at the beginning of each survey.

## 4.2 Prospects

Each prospect has been described as drawing a ball at random from an urn containing 90 coloured balls, each of which is either red or blue. The draw would be made tomorrow at noon, and the payoff depends on the colour of the ball that has been drawn with red paying the higher, and blue the lower amount. There were two possible *payoff pairs*: $(600, 100)$ and $(400, 300)$. There were also three possible *sources* of uncertainty represented by varying information on color composition in the urn: the RISK source, half of the balls are blue and half are red. In the UNCERTAINTY source, the composition of colors in the urn is unknown. In the PARTIAL source, it is known that there are 30 blue, 30 red and 30 balls in an unknown colour (red or blue).

### 4.3 Design

Subjects were randomly assigned to one of three groups differing in the types of prospects they had to evaluate:

a. source: RISK, payoff pairs: $(600, 100)$, $(400, 300)$,

b. source: UNCERTAINTY, payoff pairs: $(600, 100)$, $(400, 300)$,

c. sources: RISK, PARTIAL, UNCERTAINTY, payoff pair: $(600, 100)$.

There were 2 prospects in groups a. and b. and three prospects in group c. A subject received two MPLs per prospect, one to elicit buying and no-buying price and one to elicit short-selling and no short-selling price. The exact instructions provided to the participants, the design of the MPL table and the details of the comprehension quiz are all provided in the Appendix.

### 4.4 Subjects

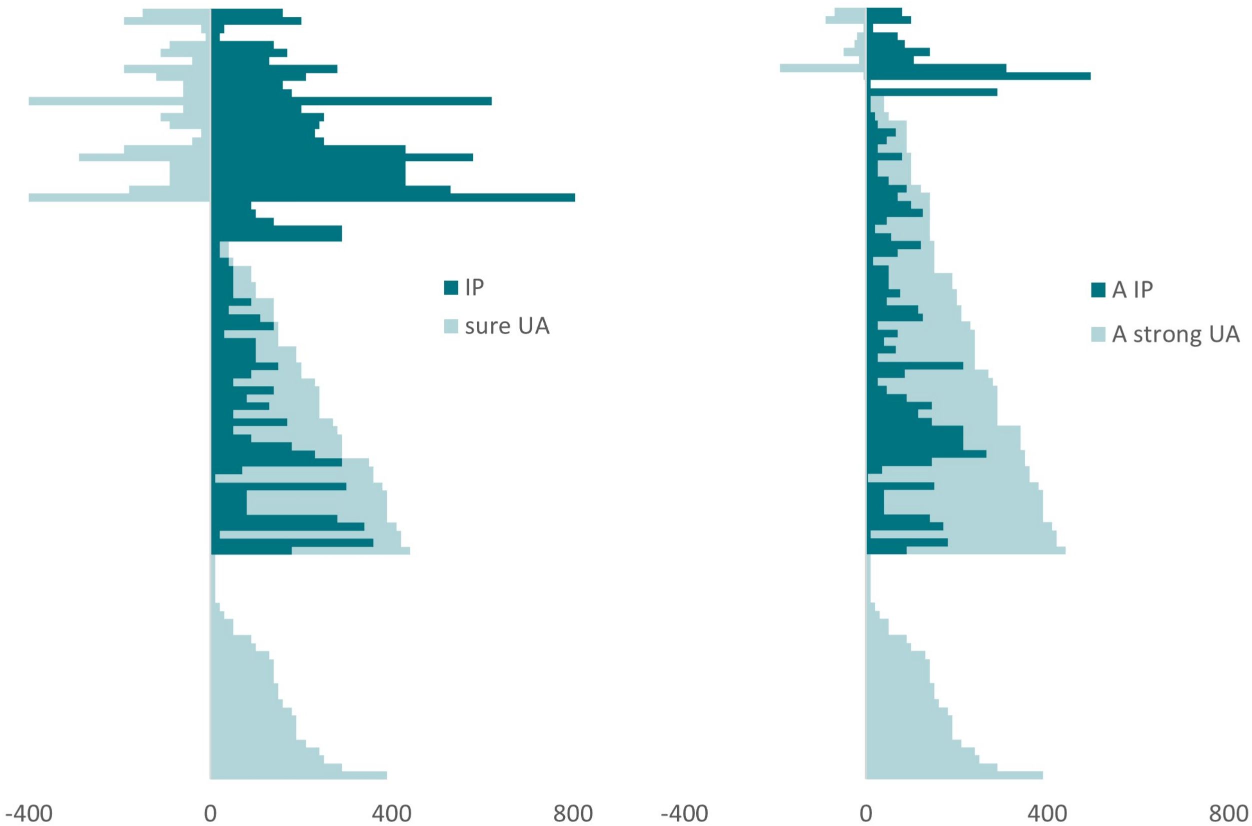
The participants in the experiment were 92 undergraduate and master students at the age ranging from 19 to 33 from the Warsaw School of Economics. The participants were not paid. Taking part was voluntary but it was explained that participation might foster understanding of some ideas in classes of decision theory.

### 4.5 Data

In total, we collected information for 207 respondent-prospect pairs. We used the following data cleaning procedure. First, we removed observations that were not complete and lacked some of the prices (e.g., due to the respondent ending the survey prematurely). However, we retained observations that did not contain the results of the Berlin numeracy test. After this step, 170 pairs were available.

The short-selling price was no greater than the buying price for 45 of these pairs. These were counted as non-standard behavior. We focused on 125 respondent-prospect pairs (coming from the total of 63 respondents) for which the short-selling price was greater than the buying price. See Table **??** for the structure of information available.

Figure 5: Absolute decompositions




### 4.6 Results

We present absolute decompositions for prospects with payoffs $(600, 100)$ in Figure 5. The sure UA decomposition appears in the left panel. There are two strong UA decompositions. The right panel presents an averaged version[7] of the two strong UA decomposition. We also present the relative decomposition for all prospects in Figure 6. The strong UA decompositions have also been averaged to obtain a single decomposition.

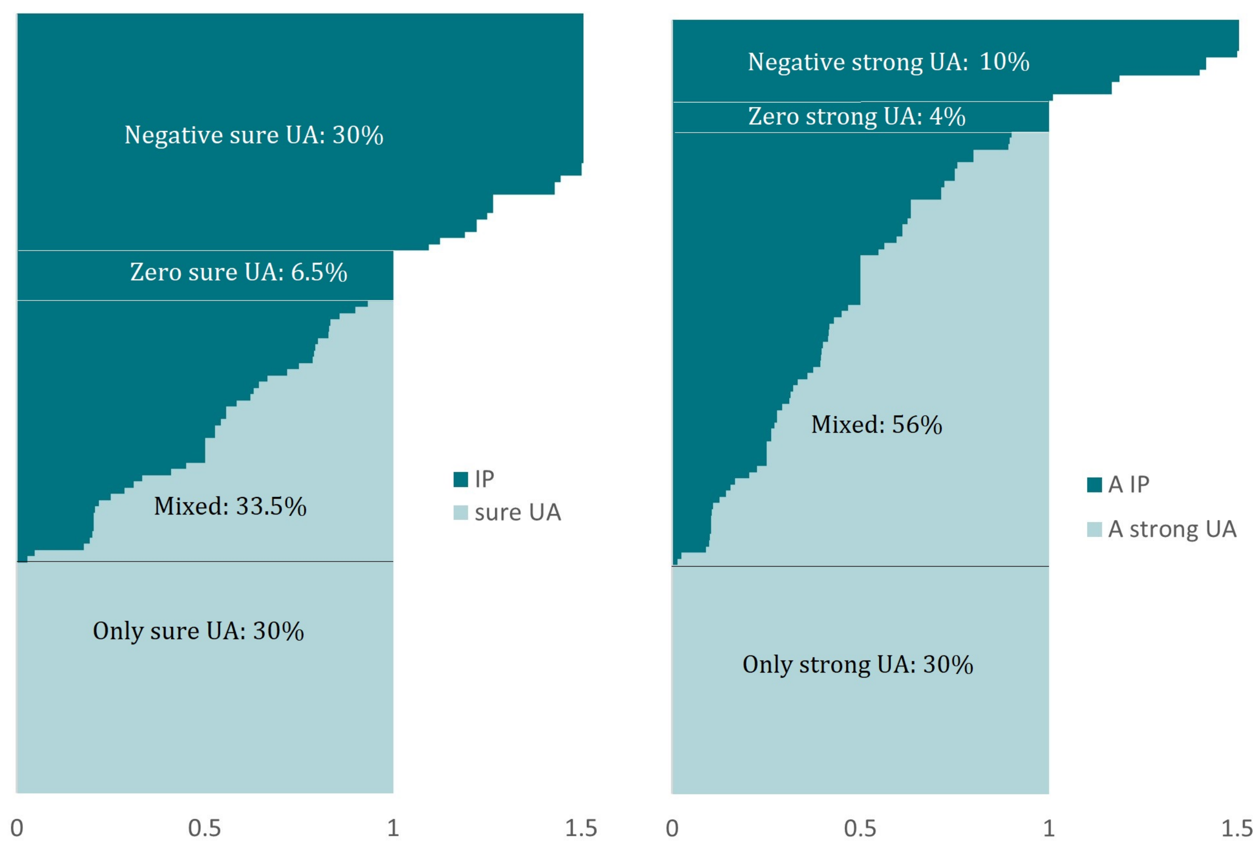
## 5 Discussion

### 5.1 Selling vs. short-selling prices

The WTA of a prospect can be understood in two ways. One stipulates that the decision maker initially has the right to a prospect $f$ and waives it in exchange for a certain amount of money.

[7]The two strong UA decompositions are not very different from each other, so averaging them out does not change qualitatively the results.

Figure 6: Relative decompositions



This method corresponds to the classical understanding of the *selling price*. The second method assumes that the decision maker issues a coupon for a prospect $f$ to a third party in exchange for a certain amount of money. This means that the decision maker assumes the obligation to pay the amounts specified by $f$ to the third party. This way of understanding WTA from the perspective of the organizer rather than the lottery participant, common in the literature on risk measures and insurance premiums (Bühlmann, 1970, p.86), is similar to the idea of taking a short position. We call the corresponding price a *short-selling price* (Eisenberger and Weber, 1995, p.224). The difference between a selling price and a short-selling price lies in the initial wealth position of the decision maker. In the selling task, the decision maker originally owns a prospect. In the short-selling task, the decision maker does not originally own the prospect. Formally, for a prospect $f$, we define the selling price functional[8]:

$$\text{selling price} \quad S : \mathcal{F} \to \mathbb{R} \qquad\qquad S(f) = \min\{\theta \in \mathbb{R} : \theta \succcurlyeq f\},$$

There are some advantages of using a short-selling price rather than a selling price as a measure of WTA. One is that by having the same initial position, we exclude possible wealth effects and avoid the subtle and elusive issue of reference point determination. Second, buying and short-selling tasks are exactly opposite to each other, which allows to focus on the asymmetry between gains and losses.

It is standard to focus on one uncertainty and assume all prior uncertainty is resolved. Let $w \in \mathbb{R}$ denote that part of decision maker's wealth that does not contain the prospect under consideration. Let $h$ be real-valued function on $S$ representing the decision maker's status quo wealth. We define $(\succcurlyeq_h)_h$, as the family of preference relations over prospects for different initial wealth positions $h$. Provided that they exist, we let $S(f)$, $B^*(f)$, and $B(f)$ denote the selling, short-selling and buying price of prospect $f$, respectively. They are defined as

$$S(f) = \min\{\theta \in \mathbb{R} : \ \theta - f \succcurlyeq_{w+f} 0\} \tag{20}$$

$$B^*(f) = \min\{\theta \in \mathbb{R} : \ \theta - f \succcurlyeq_{w} 0\} \tag{21}$$

$$B(f) = \max\{\theta \in \mathbb{R} : \ f - \theta \succcurlyeq_{w} 0\}. \tag{22}$$

The implicit assumption of the standard utility theory is that of consequentialism (Rubinstein,

[8]Existence and uniqueness follows from our assumptions.

2012, p. 122), which assumes that there is a single preference relation $\succcurlyeq$ over "final levels of wealth"[9] and for each initial wealth position $h$, the preferences $\succcurlyeq_h$ towards "changes with respect to $h$" are derived from $\succcurlyeq$ by: $f \succcurlyeq_h g \iff h+f \succcurlyeq h+g$. Applying his doctrine to the definitions of $B, B^*, S$ we obtain:

$$S(f) = \min\{\theta \in \mathbb{R} : \; w + \theta \succcurlyeq w + f\} \tag{23}$$

$$B^*(f) = \min\{\theta \in \mathbb{R} : \; w + \theta - f \succcurlyeq w\} \tag{24}$$

$$B(f) = \max\{\theta \in \mathbb{R} : \; w + f - \theta \succcurlyeq w\}. \tag{25}$$

In behavioral models, the doctrine of consequentialism is often abandoned in favor of an income-based model, which is the exact opposite to what the consequentialism claims. Instead of having a single preference relation over wealth positions, it postulates a single preferences over changes wrt the status quo wealth position. In the most radical version, (Schmidt et al., 2008; Birnbaum, 2018) the status quo position may be random (as is the case with the selling price). Then the family of preference relations $(\succcurlyeq_h)_h$ is given by $\succcurlyeq_h = \succcurlyeq$, for all $h$ for some $\succcurlyeq$ preference relation over prospects. This corresponds to the following definitions:

$$S(f) = B^*(f) = \min\{\theta \in \mathbb{R} : \; \theta - f \succcurlyeq 0\}$$
$$B(f) = \max\{\theta \in \mathbb{R} : \; f - \theta \succcurlyeq 0\}.$$

A less radical version (Kahneman and Tversky, 1979; Tversky and Kahneman, 1992) does not allow the reference point to be random. In the case of selling when the status quo is random, we may set the reference point equal to $w$. This would correspond to the prices defined as in (23)–(25) with $w = 0$.

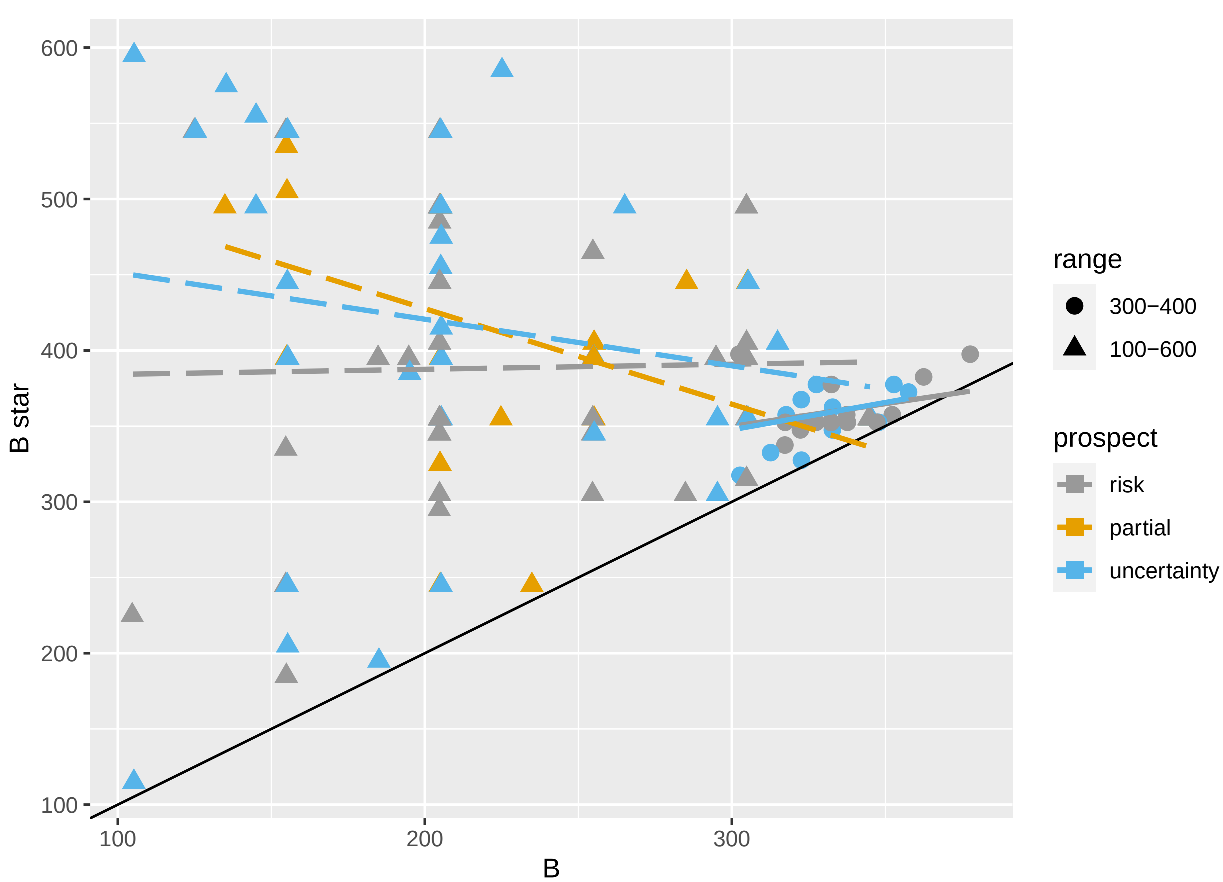
## 5.2 Related literature

**Elicitation procedures** Plott and Zeiler (2005) claim that WTA-WTP gap is due to subjects' misconceptions about the elicitation procedures. They question that the gap exists and they question its explanation based on loss aversion. Isoni et al. (2011) show that while the no-gap result holds for coffee mugs but does not hold for abstract lotteries. List and Gallet (2001); Horowitz (2006); Karni et al. (1987); Holt (1986); Segal (1988)

[9] The exact statement is: preferences over prospects whose consequences represent the decision maker's final wealth levels.

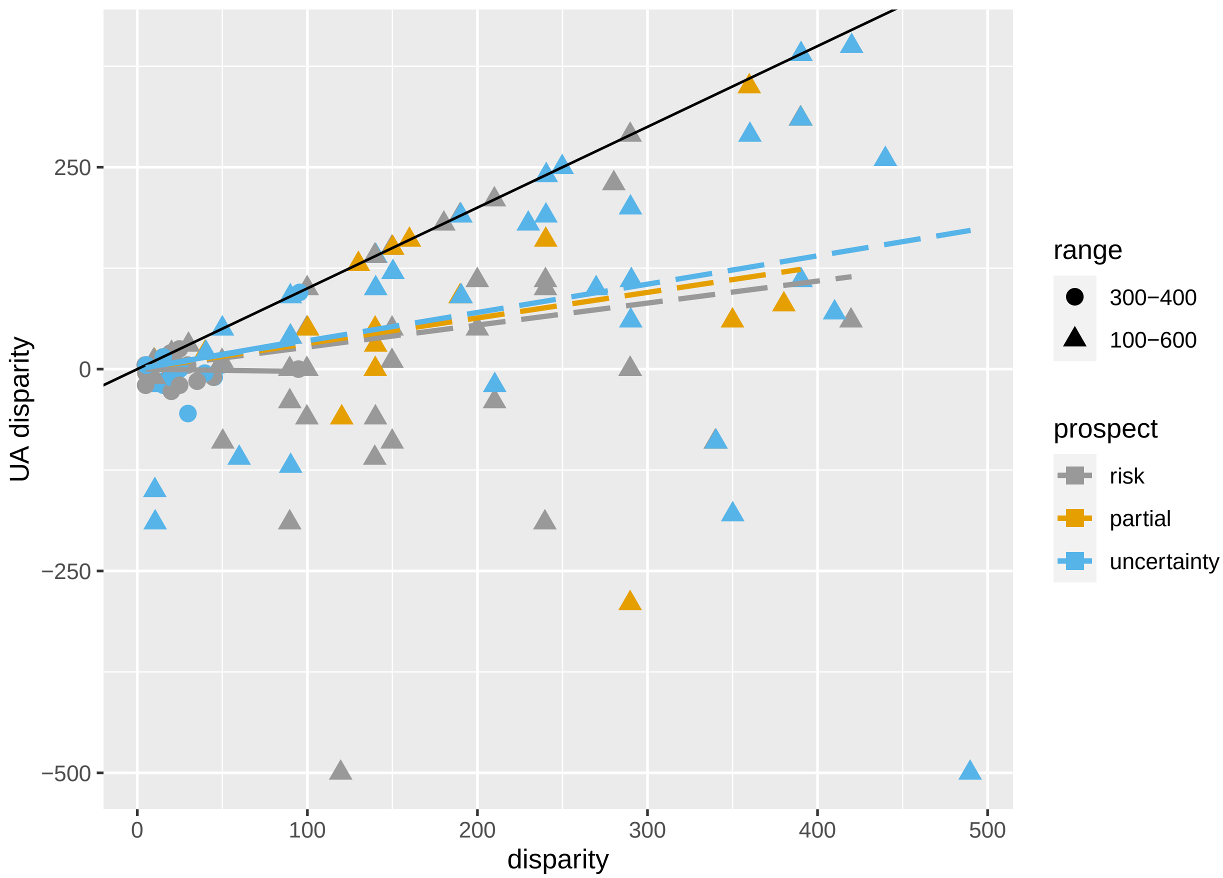
**Correlation between WTA and WTP, and correlation between the WTA-WTP gap and loss aversion** Chapman et al. (2023) WTA and WTP are weakly correlated with each other and the disparity between WTA and WTP is weakly correlated with loss aversion. This challenges the view that loss aversion is a good explanation for the disparity between WTA and WTP. We replicate their findings using our measure of WTA and our measure of loss aversion. See Table **??** for the correlation between WTA and WTP. See Table 8 for the correlation between the WTA-WTP disparity and loss aversion. Calcualate correlation, remove the black solid line from the figures

Figure 7: Correlation between WTA and WTP




**Cautious utility** Cerreia-Vioglio et al. (2024) they propose an explanation based on caution. There are some differences to our approach: First, they discuss WTA-WTP disparity in the context of the endowment effect, and hence treat WTA as a selling price of the object that I initially own. However, under their assumptions of random status quo as a reference point, there is no difference between selling and short-selling price. We take WTA as a short-selling price and hence do not study the endowment effect. Short-selling is the opposite side of the same transaction. We study

Figure 8: Total disparity vs UA disparity (loss aversion)




the asymmetry between the two opposing sides but separate it from the endowment effect, which we exclude. Tthe advantages include points 2 and 5 below. Second, they have three loss aversion measures: the lambda coefficient of the utility function, a lottery equivalent of zero, a CE of a $(x, -x; 0.5)$ prospect. They separately elicit WTA and WTP of some other prospect and then calculate the correlation. We do it smarter: a) taking short-selling insetad of selling we naturally measure the ratio of gains to losses for the same prospect for which we elicit WTA and WTP: we effectively evaluate $f$ i $-f$ (so that all gains become losses and all losses become gains). We thus can decompose into loss aversion and incompleteness. b) We do it independently from the model. For example, the short-selling and buying price definitions are very robust to changes in reference points rules and we do not place any additional restrictions. They, on the other hand, base ther conclusions on a specific model (they claculate WTA and WTP for the Kőszegi and Rabin (2007) model and the PT3 model (Schmidt et al., 2008). This is less general. c) We define uncertainty aversion as a behavioral condition related to any prospect. Their loss aversion is more vague: lambda is slope paramtere in a specific model, CE of a prospect $(x, -x; 0.5)$ is similar to risk aversion.

**WTA-WTP disparity in the cautious completion of the MUMP model** Assume $\succcurlyeq$ has a MUMP representation the set of priors and utilities is $\Phi$. Following Hara and Riella (2023), $\succcurlyeq$ is a preorder and we may consider its cautious completion $\succcurlyeq^*$ on $\mathcal{F}$, a weak order related to $\succcurlyeq$ by the axioms of consistency and caution, such that for any $f, g \in \mathcal{F}$,

$$f \succcurlyeq^* g \iff \min_{(\mu,u)\in\Phi} u^{-1}\left(\int_S u(f)d\mu\right) \geq \min_{(\mu,u)\in\Phi} u^{-1}\left(\int_S u(g)d\mu\right). \tag{26}$$

Hara and Riella (2023)[10] suggests the following interpretation: $\succcurlyeq$ represents choices that can be made with certainty, while $\succcurlyeq^*$ represents forced choices that are made even if the decision maker is not confident. For comparison we state the counterpart of Proposition 8 for the case when only forced choices are observed in the price elicitation tasks.

**Proposition 16.** *Assume that the preferences $\succcurlyeq^*$ are a cautious completion of the MUMP preferences $\succcurlyeq$ where the set of priors and utilities is $\Phi$. For any $f \in \mathcal{F}$, the following holds*

$$B(f) = B_n(f) = \min_{(\mu,u)\in\Phi} B_{\mu,u}(f),$$
$$B^*(f) = B^*_n(f) = \max_{(\mu,u)\in\Phi} B^*_{\mu,u}(f).$$

[10] See also Gilboa et al. (2010).

# A Proofs

**Proof of Proposition 1** Take any $f \in \mathcal{F}$ and fix it. We prove all statements for $B(f)$. The remaining cases are proved similar. We first show existence. If $f = \theta^*$ for some $\theta^* \in \mathbb{R}$ then by **B0**–**B1** $B(f) = \theta^* = \min f = \max f$. Assume that $f$ is nonconstant and define:

$$\begin{aligned} \mathcal{B}(f) &= \{\theta \in \mathbb{R} : f - \theta \succcurlyeq 0\}, \\ A &= \{g \in \mathcal{F} : g = f - \theta,\ \theta \in \mathbb{R}\}, \\ A' &= \{g \in \mathcal{F} : g = f - \theta,\ \theta \in \mathcal{B}(f)\}. \end{aligned}$$

We first show that $\mathcal{B}(f)$ is nonempty. Indeed it contains $\min f$: $f - \min f \geq 0$ and $f \neq \min f$, which, in view of **B1**, implies $f - \min f \succ 0$. Hence, $\min f \in \mathcal{B}(f)$. We now show that $\mathcal{B}(f)$ is bounded from above, Indeed since $f - \theta \leq 0$, $f \neq \theta$, for $\theta \geq \max f$, so by **B1** $0 \succ f - \theta$ which implies that $f - \theta \not\succcurlyeq 0$. So $\mathcal{B}(f)$ does not contain any $\theta \geq \max f$. We next show that $\mathcal{B}(f)$ is closed. $A'$ is the intersection of $A$, which is closed, and nW(0), which is also closed by **B2**. So $A'$ is also closed. Define a function $\gamma : \mathbb{R} \to \mathcal{F}$ by $\gamma(\theta) = f - \theta$. Note that $\gamma$ is a continuous function. Hence a preimagine of any closed set is closed. Note that a preimage of $A'$ is $\mathcal{B}(f)$, and since the former is closed, the latter must also be. We have shown that $\mathcal{B}(f)$ is a nonempty and closed set bounded from above. So $\mathcal{B}(f)$ contains its maximum, which proves that $B(f)$ exists. It is also unique by definition.

We now prove that $B(f) \in [\min f, \max f]$. We have already shown that $\min f \in \mathcal{B}(f)$ so by the definition of the latter $B(f) \geq \min f$. Now observe that $f - \max f \leq 0$, $f \neq \max f$, so **B1** implies that $0 \succ f - \max f$. On the other hand $f - B(f) \succcurlyeq 0$. By **B0**, $f - B(f) \succcurlyeq f - \max f$, which is the same as $f - \max f \not\succ f - B(f)$. And the latter implies by **B1** that either $f - B(f) = f - \max f$ or $f - B(f) \geq f - \max f$. This is the same as $\max f \geq B(f)$ which finishes the proof of (i).

We now show $B_n(f) \geq B(f)$. By definition $f - B(f) \succcurlyeq 0$. We consider two cases. The first case is $0 \not\succcurlyeq f - B(f)$ or $f - B(f) \succ 0$. By definition we also have $0 \succcurlyeq f - B_n(f)$. So by **B0**, $f - B(f) \succcurlyeq f - B_n(f)$, which is the same as $f - B_n(f) \not\succ f - B(f)$. Hence by **B1**, we have that $f - B(f) \geq f - B_n(f)$ or $B_n(f) \geq B(f)$. We now analyze the second case when $0 \succcurlyeq f - B(f)$, which means that $f - B(f) \sim 0$. By definition we have that $0 \succcurlyeq f - B_n(f)$, so that by **B0** we have $f - B(f) \succcurlyeq f - B_n(f)$, or equivalently, $f - B_n(f) \not\succ f - B(f)$. The latter implies by **B1** that $f - B(f) \geq f - B_n(f)$ or $B_n(f) \geq B(f)$.

We now prove the last statement. Suppose that for some prospect $f$ one of the inequalities in (5) are strict, say $B_n(f) > B(f)$. In order to show that preferences are incomplete, it suffices to show that there is pair of noncomparable prospects. Take $\theta \in \mathbb{R}$ such that $B_n(f) > \theta > B(f)$. By the definition of $B(f)$, $f - \theta \not\succcurlyeq 0$. Also, by the definition of $B_n(f)$, $0 \not\succcurlyeq f - \theta$. So 0 and $f - \theta$ are not comparable and hence $\succcurlyeq$ is incomplete.

**Proof of Proposition 2** Observe that for $X \in \{B^*, B, B_n^*, B_n\}$ satisfies translation invariance, i.e. $X(f + \lambda) = X(f) + \lambda$ for any $\lambda \in \mathbb{R}$. We show it for $X = B$ and the rest is similar:

$$B(f + \lambda) = \max\{\theta \in \mathbb{R} : f + \lambda - \theta \succcurlyeq 0\} = \lambda + \max\{\theta \in \mathbb{R} : f - \theta \succcurlyeq 0\}.$$

Moreover, for all $f \in \mathcal{F}$, the following holds: $B(-f) = -B^*(f)$. Indeed:

$$\begin{aligned} -B(-f) &= -\max\{\theta \in \mathbb{R} : -f - \theta \succcurlyeq 0\} = \min\{-\theta \in \mathbb{R} : -\theta - f \succcurlyeq 0\} = \\ &= \min\{\theta' \in \mathbb{R} : \theta' - f \succcurlyeq 0\} = B^*(f). \end{aligned}$$

Hence $B^*(f) = -B(-f) = \theta - B(\theta - f)$ and thus (6) holds. Similarly, (7) holds because $B_n$ is translation invariant and, for all $f \in \mathcal{F}$, $B_n(-f) = -B_n^*(f)$.

**Proof of Proposition 3** Suppose that UA holds. By the definition of $B$, for any nonzero prospect $f$, $f - B(f) \succcurlyeq 0$. UA implies that $B(f) - f \not\succcurlyeq 0$, which in view of the definition of $B^*$ implies that $B(f) < B^*(f)$. Assume now that $B^*(f) > B(f)$ for a nonzero prospect $f$ and assume that $f \succcurlyeq 0$ for a nonzero prospect $f$. We must prove that $-f \not\succcurlyeq 0$. By the definition of $B$ and in view of the monotonicity of $\succcurlyeq$, we have $B(f) \geq 0$ and thus $B^*(f) > 0$. From the definition of $B^*$, we obtain that $-f \not\succcurlyeq 0$. This proves the equivalence between (i) and (ii). The equivalence between (ii) and (iii) is straightforward and follows from the definition of $B^*$ and $B$.

**Proof of Proposition 4** We only prove the first part, as the second part follows similar reasoning. Suppose that the decision maker is undecided about $f$. Then there is a $\theta \in \mathbb{R}$ such that $f + \theta \not\succcurlyeq 0$ and $0 \not\succcurlyeq f + \theta$. By the definition of $B$, $-\theta > B(f)$. Similarly, by the definition of $B_n$, $B_n(f) > -\theta$. It follows that $B_n(f) > B(f)$. Similarly, if $B_n(f) > B(f)$ holds for some prospect $f$, take $\theta \in \mathbb{R}$ such that $B_n(f) > -\theta > B(f)$. By the definition of $B$ and $B_n$, it holds: $f + \theta \not\succcurlyeq 0$ and $0 \not\succcurlyeq f + \theta$. This completes the proof.

**Proof of Proposition 5** We first prove the $\Rightarrow$ part. Assume that sure UA holds and suppose, by way of contradiction, that for some nonzero prospect $f$, $B^*(f) \leq B(f)$ or $B_n^*(f) < B_n(f)$. If $B^*(f) \leq B(f)$, then take $\theta \in \mathbb{R}$ such that $B^*(f) \leq \theta \leq B(f)$. By the definitions of $B^*$ and $B$, this implies that $f - \theta \succcurlyeq 0$ and $\theta - f \succcurlyeq 0$, which implies that $0 \not\succ f - \theta$ and $0 \not\succ \theta - f$, a contradiction to sure UA. If $B_n^*(f) < B_n(f)$, then take $\theta \in \mathbb{R}$ such that $B_n^*(f) < \theta < B_n(f)$. By the definitions of $B_n$ and $B_n^*$, we have $0 \not\succcurlyeq f - \theta$ and $0 \not\succcurlyeq \theta - f$. This implies $0 \not\succ f - \theta$ and $0 \not\succ \theta - f$, a contradiction to sure UA. This finishes this part of the proof.

We now prove the $\Leftarrow$ part. We assume that for for all nonzero prospect $f$, $B^*(f) > B(f)$ and $B_n^*(f) \geq B_n(f)$. We take an arbitrary nonzero prospect $f$ such that $0 \not\succ f$. This means that $0 \not\succcurlyeq f$ or $f \succcurlyeq 0$. If $0 \not\succcurlyeq f$, then, by the definition of $B_n$, $B_n(f) > 0$. Hence $B_n^*(f) > 0$ and $B^*(f) > 0$, by assumption. In view of the definitions of $B_n^*$ and $B^*$, we obtain $0 \succcurlyeq -f$ and $-f \not\succcurlyeq 0$ or $0 \succ -f$. If $f \succcurlyeq 0$, then, by the definition of $B$, $B(f) \geq 0$, so, by assumption, $B^*(f) > 0$ and $B_n^*(f) \geq 0$. In view of the definitions of $B^*$ and $B_n^*$, we obtain $-f \not\succcurlyeq 0$ and $0 \succcurlyeq -f$, or $0 \succ -f$. This finishes the proof.

**Proof of Proposition 6** Suppose that strong UA holds and consider an arbitrary nonzero prospect $f$. By the definition of $B$, $f - B(f) \succcurlyeq 0$, which implies, by strong UA, $0 \succ B(f) - f$ or $0 \succcurlyeq B(f) - f$ and $B(f) - f \not\succcurlyeq 0$. By the definition of $B_n^*$ and $B^*$, these imply $B_n^*(f) \geq B(f)$ and $B^*(f) > B(f)$. As $f$ is arbitrary, the same holds for $-f$ and hence, in view of Proposition 2, we have $-B_n(f) \geq -B^*(f)$ or $B^*(f) \geq B_n(f)$. This finishes the proof of the first part of the proposition. To prove the converse, we take an arbitrary nonzero prospect $f$ and assume that $B_n^*(f) \geq B(f)$ and $B^*(f) > B(f)$ holds. We also assume that $f \succcurlyeq 0$. This, by the definition of $B$ implies that $B(f) \geq 0$. By our assumptions it implies that $B^*(f) > 0$ and $B_n^*(f) \geq 0$ and, by the definitions of $B^*$ and $B_n^*$, implies that $0 \succcurlyeq -f$ and $-f \not\succcurlyeq 0$, or $0 \succ -f$. This completes the proof.

**Proof of Proposition 7** Let $A$ be a $\frac{1}{2}$-probability event and let $x, y \in \mathbb{R}$. Then $(x-\theta, y-\theta; A) \sim (y-\theta, x-\theta; A)$ for all $x, y, \theta \in \mathbb{R}$. Take $\theta = B(x, y; A)$. Then $(x-\theta, y-\theta; A) \succcurlyeq 0$ and by transitivity $(y-\theta, x-\theta; A) \succcurlyeq 0$. Hence $B(y, x; A) \geq \theta = B(x, y; A)$. Repeating the same argument with $\theta = B(y, x; A)$ shows that $B(x, y; A) \geq B(y, x; A)$, which together with the previous inequality yields $B(x, y; A) = B(y, x; A)$. Similarly, one can show that $B_n(x, y; A) = B_n(y, x; A)$. Applying Proposition 2 and the already proved part, we get

$$B^*(x, y; A) - B_n^*(x, y; A) = x + y - B(y, x; A) - x - y + B_n(y, x; A) = B_n(x, y; A) - B(x, y; A).$$

**Proof of Proposition 8** We only prove it for $B$ as the rest is similar. Take an arbitrary $f \in \mathcal{F}$. We can rewrite the definition of $B$ as follows

$$B(f) = \max \left\{ \theta \in \mathbb{R} : \sum_{s \in S} \mu(s) u(f(s) - \theta) \geq 0 \quad \text{for all } (\mu, u) \in \Phi \right\}. \tag{27}$$

We will prove that $B(f) = \hat{\theta} := \min_{(\mu,u)\in\Phi} B_{\mu,u}(f)$. Note that

$$\sum_{s \in S} \mu(s) u(f(s) - \hat{\theta}) \geq 0 \quad \text{for all } (\mu, u) \in \Phi. \tag{28}$$

Hence, by (27), $B(f) \geq \hat{\theta}$. Suppose that $\theta' > \hat{\theta}$ and let

$$(\mu^*, u^*) := \arg \min_{(\mu,u)\in\Phi} B_{\mu,u}(f). \tag{29}$$

Then, by monotonicity

$$\sum_{s \in S} \mu^*(s) u^*(f(s) - \theta') < 0,$$

but this, in view of (27), implies that $\theta' \neq B(f)$. So it must be that $B(f) = \hat{\theta}$.

**Proof of Proposition 9** Let $f = (x, y; A)$ be a prospect where $x, y \in \mathbb{R}$ and $A$ is a $\frac{1}{2}$-probability event. According to (12) and (13), for any $(\mu, u) \in \Phi$, we have

$$\frac{1}{2} u(x - B_{\mu,u}(f)) + \frac{1}{2} u(y - B_{\mu,u}(f)) = 0$$
$$\frac{1}{2} u(B^*_{\mu,u}(f) - x) + \frac{1}{2} u(B^*_{\mu,u}(f) - y) = 0$$

Thus for any $(\mu, u) \in \Phi$, $B^*_{\mu,u}(f) = x + y - B_{\mu,u}(f)$. So

$$\arg \min_{(\mu,u)\in\Phi} B^*_{\mu,u}(f) = \arg \max_{(\mu,u)\in\Phi} B_{\mu,u}(f).$$

and hence, in view of Proposition 8

$$B^*_n(f) - B_n(f) = \min_{(\mu,u)\in\Phi} B^*_{\mu,u}(f) - \max_{(\mu,u)\in\Phi} B_{\mu,u}(f) = \min_{(\mu,u)\in\Phi} \left( B^*_{\mu,u}(f) - B_{\mu,u}(f) \right).$$

**Proof of Proposition 10** Assume that $\succcurlyeq$ is surely uncertainty averse. By monotonicity, UA for sure implies that for any nonzero prospect $f$, $0 \succ f$ or $0 \succ -f$. Take any $x \neq 0$ and a $\frac{1}{2}$ probability event $A$. Then $S \setminus A$ is also a $\frac{1}{2}$ probability event. Set $f = (x, -x; A)$. Then $-f = (-x, x; A)$ and by the definition of a $\frac{1}{2}$ probability event $f \sim -f$. By transitivity (**B0**), $0 \succcurlyeq f \iff 0 \succcurlyeq -f$ and $f \not\succcurlyeq 0 \iff -f \not\succcurlyeq 0$. Hence $0 \succ f \iff 0 \succ -f$ ad therefore $0 \succ f$ and $0 \succ -f$. Since $-f = (-x, x; A) = (x, -x; S \setminus A)$, we have proved that $0 \succ (x, -x; A)$ and $0 \succ (x, -x; S \setminus A)$. Because $x$ and $A$ were arbitrary, the proof of the first implication is completed. The proof of the second implication is similar. The only difference is that by transitivity, if $f \sim -f$, then $f \not\succcurlyeq 0 \iff -f \not\succcurlyeq 0$.

**Proof of Lemma 1** Assume that MUMP holds with $\Phi = \{(\mu, u) : \ u \in \mathcal{U}, \ \mu \in \Pi^u\}$ being a set of prior-utility pairs.

We first prove (i). Let $A$ be a $\frac{1}{2}$-probability event. By Definition, for any $x, y \in X$, $(x, y; A) \sim (y, x; A)$. In view of definition 5, this is equivalent to $\mu(A)u(x) + (1 - \mu(A))u(y) = \mu(A)u(y) + (1 - \mu(A))u(x)$ or $(u(x) - u(y))(2\mu(A) - 1) = 0$, for every $u \in \mathcal{U}$ and $\mu \in \Pi^u$, $u \in \mathcal{U}$. Since each $u \in \mathcal{U}$ is strictly increasing, this is equivalent to $\mu(A) = \frac{1}{2}$ for all $\mu \in \Pi^u$, $u \in \mathcal{U}$.

We now prove (ii). $\succcurlyeq$ is loss averse if for any $x \in X \setminus \{0\}$ and any $\frac{1}{2}$-probability event $A$, it holds $0 \succ (x, -x; A)$ ,or equivalently $0 \succcurlyeq (x, -x; A)$ and $(x, -x; A) \not\succcurlyeq 0$. By Definition 5 and part (i) this is equivalent to $\frac{1}{2}u(x) + \frac{1}{2}u(-x) \leq 0$ or $u(x) \leq -u(-x)$ for all $x \in X \setminus \{0\}$ and all $u \in \mathcal{U}$ and $\frac{1}{2}u(x) + \frac{1}{2}u(-x) < 0$ or $u(x) < -u(-x)$ for all $x \in X \setminus \{0\}$ and some $u \in \mathcal{U}$. The proof of (iii) is similar to that of (ii) and hence omitted.

**Proof of Proposition 11** Assume that $\succcurlyeq$ has a MUMP representation with $\Phi = \{(\mu, u) : \ u \in \mathcal{U}, \ \mu \in \Pi^u\}$ being a set of prior-utility pairs.

We first prove (i). Assume that $\succcurlyeq$ is not loss-loving. Take an arbitrary prospect $f$ and assume that $f \succcurlyeq 0$. This, under MUMP, is equivalent to $0 \leq \int_S u(f) d\mu$ for all $(\mu, u) \in \Phi$. Since $\succcurlyeq$ is not loss-loving, so by part (iii) of Lemma 1, $u(x) < -u(-x)$ holds for all $x \in X \setminus \{0\}$ and some $u \in \mathcal{U}$. So, for some $u \in \mathcal{U}$ we have $\int_S u(-f) d\mu < 0$ for all $\mu \in \Pi^u$, which is equivalent to $-f \not\succcurlyeq 0$. This proves that $\succcurlyeq$ is uncertainty averse.

We now prove (ii). Assume that $\succcurlyeq$ is loss averse. Take an arbitrary prospect $f$ and assume that $f \succcurlyeq 0$. This, under MUMP, is equivalent to $0 \leq \int_S u(f) d\mu$ for all $(\mu, u) \in \Phi$. Since $\succcurlyeq$ is loss-averse, so by part (ii) of Lemma 1, $u(x) \leq -u(-x)$ holds for all $x \in X \setminus \{0\}$ and all $u \in \mathcal{U}$. So, for any $u \in \mathcal{U}$, we have $\int_S u(-f) d\mu \leq 0$ for all $\mu \in \Pi^u$ which, under MUMP, is equivalent to $0 \succcurlyeq -f$. Moreover, since $\succcurlyeq$ is loss-averse, so by part (ii) of Lemma 1, $u(x) < -u(-x)$ holds for all $x \in X \setminus \{0\}$

and some $u \in \mathcal{U}$. So, for some $u \in \mathcal{U}$, we have $\int_S u(-f)d\mu < 0$ for all $\mu \in \Pi^u$, which, under MUMP, is equivalent to $-f \not\succcurlyeq 0$. Hence, $0 \succcurlyeq -f$ and $-f \not\succcurlyeq 0$, which is equivalent to $0 \succ -f$. This proves that $\succcurlyeq$ is strongly uncertainty averse.

**Proof of Proposition 12** We first prove the "only if" direction. Consider an arbitrary nonzero prospect $f$. By the definition of $B_1$ and $B_1^*$, agent 1 accepts each of the prospects $f - B_1(f)$ and $B_1^*(f) - f$. Set $\epsilon = B_1^*(f) - B_1(f)$. If agent 1 is more uncertainty averse than agent 2, then there exist $\delta \in \mathbb{R}$, such that agent 2 accepts $f - B_1(f) - \delta$ and $\delta + \epsilon - [f - B_1(f)] = \delta + B_1^*(f) - f$, that is

$$f - B_1(f) - \delta \succcurlyeq_2 0$$
$$\delta + B_1^*(f) - f \succcurlyeq_2 0$$

By the definition of $B_2$ and $B_2^*$, we have

$$B_1(f) + \delta \leq B_2(f)$$
$$\delta + B_1^*(f) \geq B_2^*(f)$$

Combining the two we obtain $B_1^*(f) - B_1(f) \geq B_2^*(f) - B_2(f)$. Since $f$ was arbitrary, this finishes the proof of the "only if" part of the proposition. We now prove the "if" part. Assume (16). We must prove that agent 1 is more uncertainty averse than agent 2. Take an arbitrary prospect $f$ and $\epsilon \in \mathbb{R}$ such that $f \succcurlyeq_1 0$ and $\epsilon - f \succcurlyeq_1 0$. By the definition of $B_1$ and $B_1^*$, we obtain $B_1(f) \geq 0$ and $B_1^*(f) \leq \epsilon$. So $B_1^*(f) - B_1(f) \leq \epsilon$. Taking into account (16) yields

$$B_2^*(f) - B_2(f) \leq \epsilon. \tag{30}$$

By the definition of $B_2$, we have $f - B_2(f) \succcurlyeq_2 0$. Setting $\delta = B_2(f)$, we get $f - \delta \succcurlyeq_2 0$. Thus, in view of (30), we obtain:

$$B_2^*(f) = B_2(f) + B_2^*(f) - B_2(f) \leq \delta + \epsilon,$$

which by the definition of $B_2^*$ yields $\delta + \epsilon - f \succcurlyeq_2 0$. Since $f$ was arbitrary, the proof is finished.

**Proof of Proposition 13** The proof is similar to the proof of Proposition 12 and hence omitted.

**Proof of Proposition 14** We need to prove that each of the two, (i) and (ii), implies that (18)–(19) hold whenever $g$ is more uncertain than $f$. As the proofs in the two cases, (i) and (ii), are very similar, we will proceed with one proof and highlight the differences in the two cases. Take two prospects $f, g$ such that $g$ is more uncertain than $f$, i.e. $h := g - f$ is a nonconstant prospect comonotonic with $f$. We observe that, for any $\theta \in \mathbb{R}$, $g - B_n(f) - \theta$ is more uncertain than $f - B_n(f)$ and $g - B_n(f) - \theta - (f - B_n(f)) = h - \theta$. Similarly, since $-g$ is more uncertain than $-f$ whenever $g$ is more uncertain than $f$, we note that, for any $\theta \in \mathbb{R}$, $B_n^*(f) + \theta - g$ is more uncertain than $B_n^*(f) - f$ and $B_n^*(f) + \theta - g - (B_n^*(f) - f) = \theta - h$.

Observe that for $\theta$ in the set

$$\{\theta \in \mathbb{R} : \; h - \theta \not\succ 0 \;\wedge\; \theta - h \not\succ 0\}, \tag{31}$$

$f - B_n(f)$ uncertainty-dominates $g - B_n(f) - \theta$ and $B_n^*(f) - f$ uncertainty-dominates $B_n^*(f) + \theta - g$. Similarly, for $\theta$ in the set

$$\{\theta \in \mathbb{R} : \; h - \theta \not\succcurlyeq 0 \;\wedge\; \theta - h \not\succcurlyeq 0\}, \tag{32}$$

$f - B_n(f)$ strongly uncertainty-dominates $g - B_n(f) - \theta$ and $B_n^*(f) - f$ strongly uncertainty-dominates $B_n^*(f) + \theta - g$. So, for $\theta$ in the corresponding set and monotonicity with respect to the corresponding dominance, uncertainty-dominance in the case of (i) and strong-uncertainty dominance in the case of (ii), would imply $f - B_n(f) \succcurlyeq g - B_n(f) - \theta$ and $B_n^*(f) - f \succcurlyeq B_n^*(f) + \theta - g$, which in view of the definitions of $B_n(f)$ and $B_n^*(f)$ as well as transitivity of $\succcurlyeq$, yields $0 \succcurlyeq g - B_n(f) - \theta$ and $0 \succcurlyeq B_n^*(f) + \theta - g$. By the definitions of $B_n(g)$ and $B_n^*(g)$, we would thus get $B_n(g) \leq B_n(f) + \theta$ and $B_n^*(g) \geq B_n^*(f) + \theta$, or, after combining the two inequalities, $B_n^*(g) - B_n(g) \geq B_n^*(f) - B_n(f)$. So, in order to prove that (19) is implied by (i), respectively (ii), we need to show that the set defined by (31), respectively by (32), is nonempty.

Similarly, we observe that for any $\theta \in \mathbb{R}$, $g - B(g)$ is more uncertain than $f + \theta - B(g)$ and $g - B(g) - (f + \theta - B(g)) = h - \theta$, as well as $B^*(g) - g$ is more uncertain than $B^*(g) - \theta - f$ and $B^*(g) - g - (B^*(g) - \theta - f) = \theta - h$. So for $\theta$ in the set defined by (31), $f + \theta - B(g)$ uncertainty-dominates $g - B(g)$ and $B^*(g) - \theta - f$ uncertainty-dominates $B^*(g) - g$. Similarly, for $\theta$ in the set defined by (32), $f + \theta - B(g)$ strongly uncertainty-dominates $g - B(g)$ and $B^*(g) - \theta - f$ strongly uncertainty-dominates $B^*(g) - g$. So, for $\theta$ in the corresponding set and monotonicity with respect to the corresponding dominance, uncertainty-dominance in the case of (i) and strong-uncertainty dominance in the case of (ii), would imply $f + \theta - B(g) \succcurlyeq g - B(g)$ and $B^*(g) - \theta - f \succcurlyeq B^*(g) - g$, which in view of the definitions of $B(f)$ and $B^*(f)$ as well as transitivity of $\succcurlyeq$, yields $f+\theta-B(g) \succcurlyeq 0$ and $B^*(g)-\theta-f \succcurlyeq 0$. By the definitions of $B(g)$ and $B^*(g)$, we would thus get $B(f) \geq B(g)-\theta$ and $B^*(f) \leq B^*(g)-\theta$, or, after combining the two inequalities, $B^*(g)-B(g) \geq B^*(f)-B(f)$. So, in order to prove that (18) is implied by (i), respectively (ii), we need to show that the set (31), respectively (32), is nonempty.

The above arguments in getting (18) and (19) rely on the sets defined by (31) and (**??**) being nonempty. It is thus left to show that this is indeed the case. In the case (i), $\succcurlyeq$ satisfies sure uncertainty aversion. Proposition 5 implies $B_n^*(h) \geq B_n(h)$ and so, there is $\theta \in \mathbb{R}$ such that $B_n(h) \leq \theta \leq B_n^*(h)$. By the definitions of $B_n^*$ and $B_n$, $0 \succcurlyeq h-\theta$ and $0 \succcurlyeq \theta-h$, and hence we have $h-\theta \not\succ 0$ and $\theta-h \not\succ 0$. This proves that the set of defined by (31) is nonempty. In the case (ii), $\succcurlyeq$ satisfies uncertainty aversion. Proposition 3 implies that $B^*(h) > B(h)$ and so, there is there is $\theta \in \mathbb{R}$ such that $B(h) < \theta < B^*(h)$. By the definitions of $B^*$ and $B$, $h-\theta \not\succcurlyeq 0$ and $\theta-h \not\succcurlyeq 0$. This proves that the set defined by (32) is nonempty and finishes the proof.

**Proof of Proposition 16** Consider an arbitrary prospect $f \in \mathcal{F}$. We only show the first statement as the second is proved similarly. The definitions of $B$ and $B_n$ are rewritten as follows

$$\begin{aligned} B(f) &= \max\left\{\theta \in \mathbb{R}:\ \min_{(\mu,u)\in\Phi} u^{-1}\left(\sum_{s\in S}\mu(s)u(f(s)-\theta)\right) \geq 0\right\}, \\ B_n(f) &= \min\left\{\theta \in \mathbb{R}:\ 0 \geq \min_{(\mu,u)\in\Phi} u^{-1}\left(\sum_{s\in S}\mu(s)u(f(s)-\theta)\right)\right\}. \end{aligned} \tag{33}$$

We will prove that $B_n(f) = \hat{\theta} := \min_{(\mu,u)\in\Phi} B_{\mu,u}(f)$. The proof that $B(f) = \hat{\theta}$ is similar and hence omitted. Applying (28) and using the fact that $u$ is strictly increasing and bijective, we obtain

$$u^{-1}\left(\sum_{s\in S}\mu(s)u(f(s)-\hat{\theta})\right) \geq 0 \quad \forall(\mu,u) \in \Phi. \tag{34}$$

Furthermore, in view of (29) and (12), we have

$$u^{*-1}\left(\sum_{s\in S}\mu^*(s)u^*(f(s)-\hat{\theta})\right) = 0, \tag{35}$$

which after combining with (34) yields:

$$\min_{(\mu,u)\in\Phi} u^{-1}\left(\sum_{s\in S}\mu(s)u(f(s)-\hat{\theta})\right)=0. \tag{36}$$

And hence, by (33), $B_n(f)\leq\hat{\theta}$. Suppose that $\theta'<\hat{\theta}$. Then, by (36) and the fact that $u$ is strictly increasing, we have (???)

$$\min_{(\mu,u)\in\Phi} u^{-1}\left(\sum_{s\in S}\mu(s)u(f(s)-\theta')\right)>0,$$

which contradicts (12). Hence $B_n(f)=\hat{\theta}$. This finishes the proof.

# B Experimental details

### Procedure

The experiment has been conducted as an online survey. The participants has been sent a link to the survey, which they could open at any moment and finish at their own pace. After opening the survey, the participants has been randomly assigned to one of the groups (A, B or C). The survey started with a comprehension test, after which the participant had been asked to complete the tasks assigned to their group. The order of the tasks has been randomized in two stages. For groups A and B, in the first stage the order of the roles (buyer, seller, issuer) has been drawn at random. The order of the payoff pairs has been drawn in the second stage. The sequence of the tasks was such that the participant would evaluate the lotteries for both payoff pairs for a single role and then move to the next role, with the same order of payoff pairs. Therefore for the order of roles buy, sell, issue and the order of payoff pairs $(600,100)$, $(400,300)$ the order of the tasks would be buy $(600,100)$, buy $(400,300)$, sell $(600,100)$, sell $(400,300)$, issue $(600,100)$, issue $(400,300)$. In group C the order of the tasks has been randomized in a similar way, with the only difference being that in the second stage of we have drawn the order of the prospects instead of the payoff pairs.

### Methods

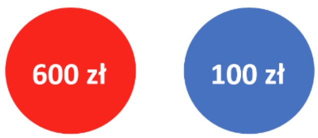
Comprehension test had two parts. In the first part, the participant has been shown the instructions, including the description of the lottery and of the roles (buying, selling, issuing), and has been asked five questions with the aim of testing their understanding of the task. The instructions given to the participant are shown in figure 9 and questions in figure 10.

Figure 9: Comprehension test: instruction

## Comprehension Quiz

Of course, people's preferences are different. As a result, most survey questions do not have one right or wrong answer. However, some answers suggest that the question has been misunderstood (e.g. if a given person declares that they would pay 100 PLN for a single 50 PLN banknote). On this screen we want to make sure you fully understand the scenarios we are asking you to imagine. Therefore, we may point out some of your answers as potentially incorrect.

Imagine an urn containing only blue and red balls. One ball will be **drawn randomly TOMORROW** at noon. **Ticket X** entitles its owner to receive a cash prize, paid right after the draw, the amount of which depends on the color of the drawn ball: the red ball pays **600 zlotys** and the blue ball pays **100 zlotys**.




We will present you with three decision scenarios, each of which takes place **TODAY**.
**Scenario 1:** Buying ticket X
You do not have ticket X, but you can buy one for a certain amount paid today.

**Scenario 2:** Selling ticket X
You already have one ticket X. You can sell it for a certain amount which you will receive today. Please note that by selling the ticket you are waiving your right to receive one of the cash prizes paid to the ticket holder tomorrow.

**Scenario 3:** Issuing ticket X
In this scenario, you act as a bank. You can issue one ticket X to another person in exchange for a certain amount paid today. Please note that by doing this you are committing to pay the ticket holder the prize determined in the draw tomorrow.

The answers to the first two questions in the test had to be correct (respectively 100 and 600) in order for the participant to be able to advance. In the other questions, the expected answers has been Yes, No and No, in that order. However, other answers to those questions did not block the participant from advancing. Instead, the participant would be shown a reassessment screen, with a clarification and an opportunity to change their answer (or provide a definite one, in case they were not sure). Example of this reassessment screen (for answers: Yes, No, I'm not sure) is provided by figure 11.

In the main part of the survey, for each task the participant has been shown two screens. The first screen provided the instructions for the specific task, and the second was a MPL table. The template for the instructions has been as follows.

(SCENARIO) a ticket PMAX-PMIN (PROSPECT DESCRIPTION)

Ticket PMAX-PMIN entitles the owner to draw one ball from the urn. The payout depends on the color of the drawn ball:

Figure 10: Comprehension test: questions

What is the minimum payout in zlotys you will receive tomorrow if you have one ticket X?

What is the maximum payout in zlotys you will have to pay tomorrow if you issue one ticket X?

If X is offered for free, would you take it?

- Yes
- No
- I'm not sure

If you had one ticket X, would you sell it for free?

- Yes
- No
- I'm not sure

Would you issue one ticket X to another person for free?

- Yes
- No
- I'm not sure

Figure 11: Comprehension test: reassessment

## Comprehension Quiz

You replied that **you didn't want ticket X for free**. This way you lose a certain profit: 600 zlotys if a red ball is drawn or 100 zlotys if a blue ball is drawn.

Please consider the same question again.

- I am changing my answer and want ticket X for free.
- I still don't want this ticket for free.

You replied that **you would sell ticket X for free**. This way you lose a certain profit: 600 zlotys if a red ball is drawn or 100 zlotys if a blue ball is drawn.

Please consider the same question again.

- I'm changing my answer and would not sell ticket X for free.
- I still want to sell ticket X for free.

You replied that **you were not sure if you wanted to issue ticket X for free**. Note that by issuing the ticket for free you lose money for sure: 600 zlotys if a red ball is drawn or 100 zlotys if a blue ball is drawn.

Please consider the same question again.

- I don’t want to issue ticket Y for free.
- I want to issue ticket Y for free.

Next

The blue ball pays PMAX PLN,

The red ball pays PMIN PLN.

(SCENARIO DESCRIPTION)

In this instruction, PMAX-PMIN is simply one of the payoff pairs, that is either 600-100 or 400-300. Prospect description depends on the source of uncertainty, namely risk, uncertainty, or partial uncertainty, and is as follows.

- (Risk): Imagine an urn containing 90 colored balls half of which are blue and half are red.
- (Uncertainty): Imagine an urn containing 90 blue or red balls of unknown proportions.
- (Partial): Imagine an urn containing 90 colored balls, each of which is either blue or red.

  You know that 30 of these balls are blue, 30 red, and 30 of an unknown color (blue or red).

Finally, scenario in the instruction corresponds to one on the three scenarios in the task, that is buying, selling or issuing a ticket. Scenario descriptions are listed below.

- (Buy): On the next screen we will present you with several possible prices and you have to answer if you would BUY the ticket at that price.
- (Sell): Imagine you hold one such ticket. On the next screen we will present you with several possible prices and you have to answer if you would SELL this ticket at that price.
- (Issue): Suppose you can issue one such ticket to another (anonymous) person. In a sense you will be acting like a bank to this person: you will have to pay out (with your own money) the cash reward to the owner of ticket (of course the result of the draw remains unknown for now).

  The other person must pay you to get the ticket. On the next screen we will present you with several possible prices and you have to answer if you would ISSUE this ticket at that price.

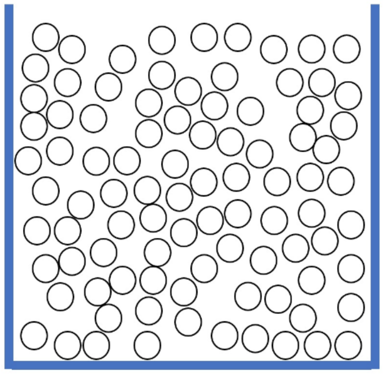
Examples of the exact instructions that were provided to the participants are shown in figure 12.

After the instructions, the participant has been shown the MPL table to choose from, as shown in figure 13. The participant did not select the values in the middle column of this table, corresponding to the prices at which they are not sure. These values has been selected automatically based on the

Figure 12: Instructions provided to the participants

# Buying a ticket *600-100*

Imagine an urn containing 90 blue or red balls of unknown proportions.

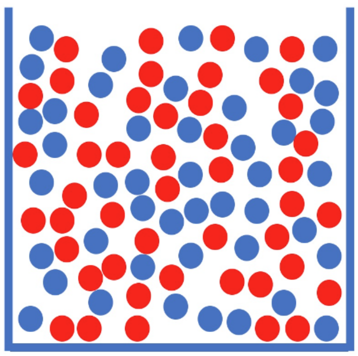


Ticket *600-100* entitles the owner to draw one ball from the urn. The payout depends on the color of the drawn ball:
**The blue ball pays 600 PLN**,
**The red ball pays 100 PLN**.

On the next screen we will present you with several possible **prices** and you have to answer **if you would BUY the ticket at that price.**

# Issuing a ticket *400-300*

Imagine an urn containing 90 colored balls **half of which are blue and half are red**.



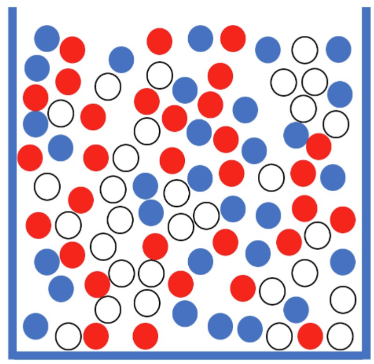
Ticket *400-300* entitles the owner to draw one ball from the urn. The payout depends on the color of the drawn ball:
**The blue ball pays 400 PLN**,
**The red ball pays 300 PLN**.

Suppose you can **issue one such ticket** to another (anonymous) person. In a sense **you will be acting like a bank** to this person: you will have to pay out (with your own money) the cash reward to the owner of ticket *400-300* (of course the result of the draw remains unknown for now).

The other person must pay you to get the ticket. On the next screen we will present you with **several** possible prices and you have to answer **if you would ISSUE this ticket at that price**.

### Selling a ticket *AMBIGUITY*

Imagine an urn containing 90 colored balls, each of which is either blue or red. You know that **30 of these balls are blue, 30 red, and 30 of an unknown color (blue or red)**.



Ticket *AMBIGUITY* entitles the owner to draw one ball from the urn. The payout depends on the color of the drawn ball:
**The blue ball pays 600 PLN**,
**The red ball pays 100 PLN**.

Imagine you hold one such ticket. On the next screen we will present you with several possible **prices** and you have to answer if **you would SELL this ticket at that price.**

selections in the left and right columns of the table. Additionally, the table enforced monotonicity, meaning that the selection of a price at which the participant certainly would buy (or sell or issue) automatically blocked the possibility of selecting a lower price as a price at which the participant certainly would not buy (higher price for sell or issue). Those prices would automatically be selected as the prices at which the consumer certainly would buy (or sell or issue). Figure 14 shows the MPL table after just two selections: 325 PLN as a price at which the participant certainly would buy, and 365 PLN as a price at which they certainly would not.

## Demographics of participants

The basic demographic information on the sample of respondents in the dataset used for analysis is provided in Table 1. What is noticeable is that by far the majority of respondents obtained high scores in Berlin numeracy test: 70.5% of respondents were classified in the 4th quartile (highest scorers) as per [reference to this approach of interpreting Berlin]

Figure 13: MPL table before selections

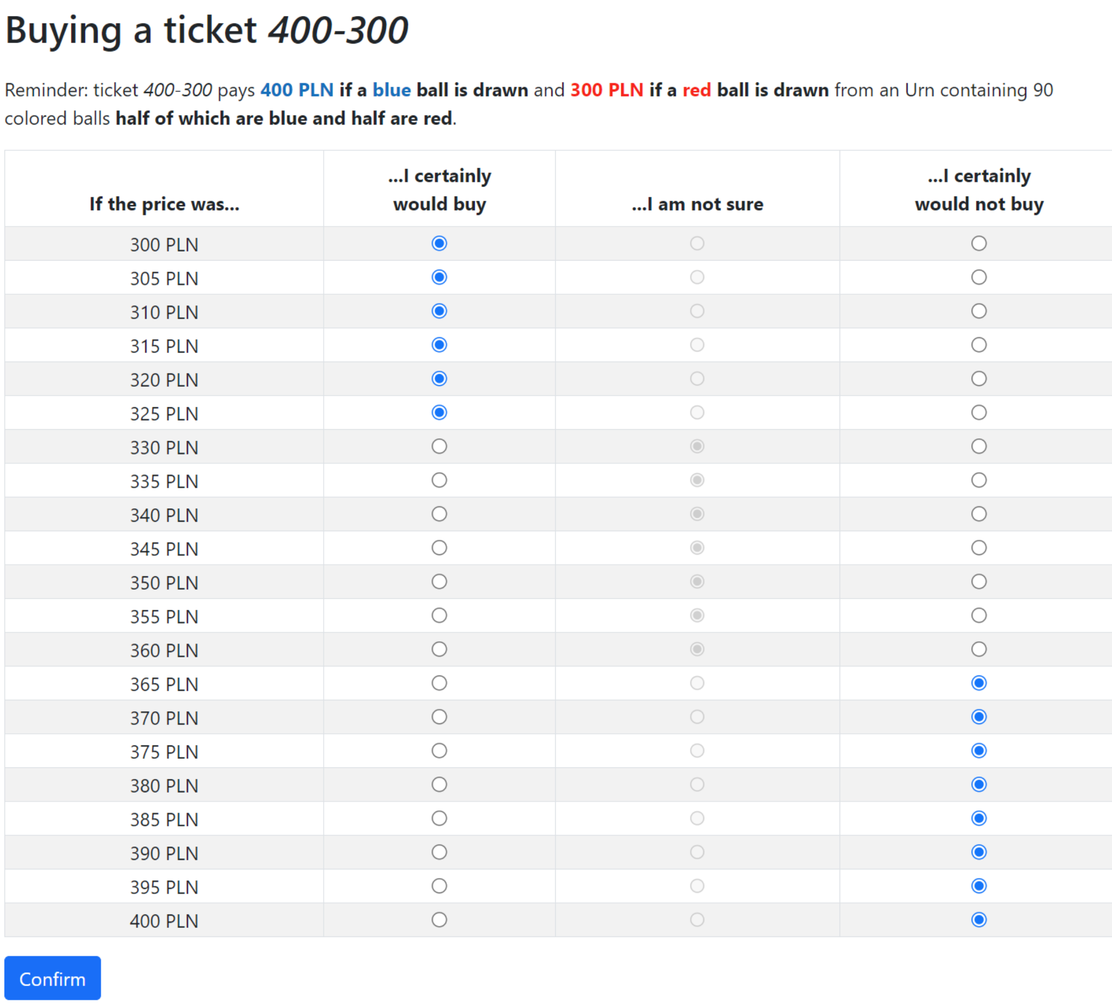
## Buying a ticket *400-300*

Reminder: ticket *400-300* pays **400 PLN if a blue ball is drawn** and **300 PLN if a red ball is drawn** from an Urn containing 90 colored balls **half of which are blue and half are red**.

| If the price was... | ...I certainly would buy | ...I am not sure | ...I certainly would not buy |
|---|---|---|---|
| 300 PLN | | | |
| 305 PLN | | | |
| 310 PLN | | | |
| 315 PLN | | | |
| 320 PLN | | | |
| 325 PLN | | | |
| 330 PLN | | | |
| 335 PLN | | | |
| 340 PLN | | | |
| 345 PLN | | | |
| 350 PLN | | | |
| 355 PLN | | | |
| 360 PLN | | | |
| 365 PLN | | | |
| 370 PLN | | | |
| 375 PLN | | | |
| 380 PLN | | | |
| 385 PLN | | | |
| 390 PLN | | | |
| 395 PLN | | | |
| 400 PLN | | | |

Confirm

Figure 14: MPL table with selections



# Buying a ticket *400-300*

Reminder: ticket *400-300* pays **400 PLN if a blue ball is drawn** and **300 PLN if a red ball is drawn** from an Urn containing 90 colored balls **half of which are blue and half are red**.

| **If the price was...** | **...I certainly would buy** | **...I am not sure** | **...I certainly would not buy** |
|---|---|---|---|
| 300 PLN | ◉ | ○ | ○ |
| 305 PLN | ◉ | ○ | ○ |
| 310 PLN | ◉ | ○ | ○ |
| 315 PLN | ◉ | ○ | ○ |
| 320 PLN | ◉ | ○ | ○ |
| 325 PLN | ◉ | ○ | ○ |
| 330 PLN | ○ | ◉ | ○ |
| 335 PLN | ○ | ◉ | ○ |
| 340 PLN | ○ | ◉ | ○ |
| 345 PLN | ○ | ◉ | ○ |
| 350 PLN | ○ | ◉ | ○ |
| 355 PLN | ○ | ◉ | ○ |
| 360 PLN | ○ | ◉ | ○ |
| 365 PLN | ○ | ○ | ◉ |
| 370 PLN | ○ | ○ | ◉ |
| 375 PLN | ○ | ○ | ◉ |
| 380 PLN | ○ | ○ | ◉ |
| 385 PLN | ○ | ○ | ◉ |
| 390 PLN | ○ | ○ | ◉ |
| 395 PLN | ○ | ○ | ◉ |
| 400 PLN | ○ | ○ | ◉ |

Confirm

Table 1: Summary of sample characteristics

| Characteristic | number (%) |
|---|---|
| *Gender* [1] | |
| female | 34 (27.2%) |
| male | 85 (68.0%) |
| other | 4 (3.2%) |
| missing | 2 (1.6%) |
| *Age* [1] *(mean and standard deviation)* | |
| age | 22.1 (2.3) |
| *Survey comprehension* | |
| 1 [???] | 106 (84.8%) |
| 2 [???] | 18 (14.4%) |
| 3 [???] | 1 (0.8%) |
| *Berlin numeracy test* | |
| 1 | 11 (9%) |
| 2 | 17 (13.9%) |
| 3 | 8 (6.6%) |
| 4 | 86 (70.5%) |
| *Education* | |
| sse [???] | ... |

[1] in case we need any

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